

# Assessing the international spillovers between the US and the euro area : evidence from a two-country DSGE-VAR.\*

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## Abstract

This paper evaluates the size of international spillovers between the US and the euro area from a structural perspective, applying the DSGE-VAR methodology developed by [Del Negro and Schorfheide \(2004\)](#). The authors propose an interesting metric to evaluate the potential misspecifications of DSGE models and indirectly provide a new class of structural Bayesian VAR: their approach uses the DSGE model to shape the prior odds for a Bayesian VAR and provide an identification scheme consistent with the theoretical model. In this paper, we extend this research agenda to a two-country framework for the euro area and the US. We assess the international spillovers of structural shocks through the DSGE-VAR estimation while highlighting the potential shortcomings of commonly used open-economy models. We systematically compare the properties of the DSGE-VAR and the DSGE along different dimensions, from conditional moments, shocks decompositions of moments to impulse response functions. Our results show in particular that some important misspecifications are present in the international transmission of shocks and that some common factors could be needed. The demand spillovers are not reinforced in the DSGE-VAR while the international propagation of US monetary policy shock has a stronger positive spillover on the euro area.

*Keywords:* DSGE models, New open economy macroeconomics, Bayesian estimation.

*JEL classification:* E4, E5, F4.

## 1 Introduction

The main objective of this paper is to evaluate the size of international spillovers between the US and the euro area from a structural perspective, applying the DSGE-VAR methodology developed by [Del Negro and Schorfheide \(2004\)](#).

Several papers have been estimating two-country DSGE models (including [Adjemian et al. \(2007\)](#), [De Walque et al. \(2005\)](#), [Rabanal and Tuesta \(2006\)](#), [Bergin \(2006\)](#) or [Adolfson et al. \(2005\)](#)). Most of those

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studies report limited international interdependence and show a limited success in matching key statistics of international business cycles. Evidence indeed suggests that some structural relations driving the international transmission in workhorse open-economy structural models may be poorly supported by the data.

Beyond the structural modeling of cross-country spillovers, an extensive econometric literature has investigated the empirical evidence on the international transmission of shocks. Many studies in particular have explored the international spillovers between the US and G7 countries using structural vector autoregressions and deliver a quite diverse set of results. [Stock and Watson \(2003\)](#) for example find a dominant role of common international shocks in explaining US-G7 co-movements while the effects of spillovers from idiosyncratic shocks are limited. Conversely, [Kim \(2001\)](#) for example finds evidence of strong spillovers from the US to G7 countries.

At the same time, [Del Negro and Schorfheide \(2004\)](#) propose an interesting metric to evaluate the potential misspecifications of DSGE models and indirectly provide a new class of structural Bayesian VAR: their approach uses the DSGE model to shape the prior odds for a Bayesian VAR and provide an identification scheme consistent with the theoretical model. In this set-up, the optimal weight on the DSGE model for the BVAR priors as well as the comparison of impulse responses between the structural BVAR (or DSGE-VAR) and the DSGE constitute key dimensions to assess the relevance of the economic restrictions embodied in the structural model. [Del Negro et al. \(2006\)](#) applied this approach to a medium-scale closed-economy DSGE for the US economy. In this paper, we extend this research agenda to a two-country framework for the euro area and the US. We assess the international spillovers of structural shocks through the DSGE-VAR estimation while highlighting the potential shortcomings of commonly used open-economy models.

In this paper, we take a radical stance and investigate the international transmission of structural shocks in a framework which abstracts from common sources of fluctuations. Obviously, there is a fundamental misspecification in our modeling framework since we use only a two-country set-up, abstracting from third-country interactions, and because data support, if not the dominance of common factors, but at least their significant contribution. Nevertheless, our analysis aims at giving a chance for our structural

description of the US-euro area interactions to meet the data and highlight potential avenues for further advances in international modeling . For a more technical reason, note that the identification scheme of the DSGE-VAR approach limits the number of structural shocks to the number of observed variables and it is *ex ante* difficult to replace any of the idiosyncratic shocks by a common one.

As in [Adjemian et al. \(2007\)](#), we use an explicit two-country US-euro area framework that allows for estimating and testing structural differences across the two areas. In contrast to the small open economy specification of [Adolfson et al. \(2005\)](#), it also allows for two-way economic and financial interaction between the two areas. The model shares many features common in open-economy DSGE models. Exchange rate pass-through is incomplete due to some nominal rigidity in the buyer's currency. The specification is flexible enough to let the data discriminate between the polar cases of local-currency-pricing (LCP) and producer-currency-pricing (PCP). Financial markets are incomplete internationally and a risk premium on external borrowing is related to the net foreign asset position. Finally, even under flexible prices and wages, purchasing power parity does not hold due to a home bias in aggregate domestic demand. As in [Christiano et al. \(2005\)](#) we introduce a number of nominal and real frictions such as sticky prices, sticky wages, variable capital utilization costs and habit persistence. In addition, following [Smets and Wouters \(2003\)](#) a large set of structural shocks enters the model. The open economy dimension also requires additional disturbances. We add a shock to the uncovered interest rate parity condition (UIP) as it is usually done in the open economy literature, a preference shock on the relative home bias and two shocks to the distribution sector markups (affecting the CPI equations).

Obviously, the use of a two-country framework implies that the rest of the world is ignored. For comparison purposes, we also tried to stick as closely as possible to the modeling framework of [Del Negro et al. \(2006\)](#), while at the same time introducing the most important New Open Economics Macroeconomics (NOEM) features. Given the relatively simple trade structure underlying our model, we did not explicitly include bilateral export and import quantities and prices in our set of macro variables to be used in the estimation. Empirically, the transmission channels of the various shocks that work through trade quantities and prices will be captured in a reduced form by their effects on relative aggregate demand, consumer versus producer prices and the current account.

The main contributions of the paper, which, to our knowledge, constitutes the first DSGE-VAR estimation of two-country DSGE for the US and the euro area, cover several dimensions.

First, from a methodological point of view, we improved on [Del Negro et al. \(2006\)](#) by jointly estimating the posterior distribution of the parameter driving the weight to put on the DSGE in the DSGE-VAR estimation. In addition, the paper provides a natural framework to extend the work of [Del Negro et al. \(2006\)](#) to the euro area: we indeed put some emphasis on the assessment of domestic transmission of shocks for both the US and the euro area. There are clear signs of misspecification in the "closed-economy" transmission channels of our two-country model. In this respect, some of our results for the US are similar to what [Del Negro et al. \(2006\)](#) obtain and it seems that the euro area block suffers slightly more from misspecification than the US block.

Moreover, the DSGE-VAR suggests stronger economic interactions between the US and the euro area. Regarding cross-country correlations in particular, the low interdependence generated by the DSGE model compared with the DSGE-VAR may be due to fundamental misspecifications in some transmission channels as well as the structural specifications of disturbances. The DSGE-VAR estimation suggests that the demand spillovers may not have a predominant role in driving cross-country output correlation. Instead, labor supply and policy shocks for example have strong positive transmission in the DSGE-VAR. However, it is important to keep in mind that, in presence of serious model-misspecification about the international dimension, the identification scheme used in the DSGE-VAR may induce spurious transmissions. In particular, if most of the comovements between the US and the euro area were due to common factors, the DSGE-VAR estimation of the theoretical model which is based on structural shocks independent from one country to the other, would most likely generate high spillovers as some country-specific shocks would tend to capture the effects of common disturbances.

In order to assess the economic relevance of the spillovers measured by the DSGE-VAR, we systematically compare the impulse responses between the DSGE-VAR and the DSGE and present here some of the results. A first conclusion is that the international spillovers on activity from demand shocks, which theoretically should be positive and strong but remain moderate in the DSGE, are not strengthened by the DSGE-VAR estimation in the short to medium term. Second, the monetary policy contractions seem to

have a pronounced negative impact on foreign activity and in particular in the case of the US monetary policy shock which induces a substantial and protracted increase in euro area interest rate. Third we find a surprisingly strong positive spillover of euro area labor supply shock. Our analysis suggests that this spillover may not be economically relevant and that, in the identification scheme, the euro area labour supply shock may be capturing the transmission of a common supply factor. Fourth, the transmission of the UIP shock in the DSGE-VAR points to a much stronger and persistent impact on domestic demand as well as a higher pass-through on CPI and PPI inflation rates.

Finally, regarding exchange rate adjustment, it turns out that the DSGE-VAR is not strengthening substantially the role of domestic fundamentals in the determination of the nominal exchange rate. By performing a DSGE-VAR estimation of a model where the nominal exchange is exogenous, we show that the DSGE-VAR only reestablishes a weak dependence on the other variables.

The rest of the paper is organized as follows. In section [2], the theoretical model is derived. Section [3] describe the DSGE-VAR estimation methods and present the parameter estimates. Section 4 explores the structural properties of the DSGE-VAR and the DSGE, focusing on propagation of shocks as well as shock decompositions of variances and cross-country correlations.

## 2 Theoretical model

In this section, we describe the theoretical features of the two-country model that we will estimate. The specifications are very close to [Adjemian et al. \(2007\)](#). For the paper to be self-containing, the full set of structural relations derived from the decision problems are reported in the appendix while the main decision problems are exposed in this section.

The world economy is composed of two symmetric countries: *Home* and *Foreign*. In each country, there is a continuum of “single-good-firms ” indexed on  $[0, 1]$ , producing differentiated goods that are imperfect substitutes. The number of households is proportional to the number of firms. Consumers receive utility from consumption and disutility from labor. In each country, the consumption baskets

aggregating products from both countries have biased preferences towards locally produced goods.

Regarding domestic frictions, the model is mainly based on [Christiano et al. \(2005\)](#) and [Smets and Wouters \(2003\)](#). The sophistication of modelling framework is first guided by the need to match a certain level data coherence, and in this respect, available studies point to an appropriate set of necessary frictions. However, we prefer to restrain this degree of sophistication in order to better understand the normative dimensions of the model, and in particular, we do not include non-tradables in this set-up. Therefore, we introduce in the model some relevant frictions to induce intrinsic persistence in the propagation of shocks, including adjustment costs on investment and capacity utilization, habit persistence and staggered nominal wage and price contracts with partial indexation.

Concerning international frictions, we assume that financial markets are complete domestically but incomplete internationally. Moreover export prices are sticky in the producer currency for a fraction of firms and in the buyer currency for the rest.

Finally, we specify a sufficient number of structural shocks in order to account for the stochastic properties of the observed data. Compared with the closed-economy models, we introduce a risk premium shock on the uncovered interest-rate parity, a preference shock on the degree of home bias in consumption and markup shocks affecting specifically the CPI inflation rates.

For the sake of clarity, most of the derivation will be pursued for country  $H$ . Analogous relations hold for country  $F$ .

## 2.1 Households decision problem

At time  $t$ , the utility function of a generic domestic consumer  $h$  belonging to country  $H$  is

$$\mathcal{W}_t(h) = \mathbb{E}_t \left\{ \sum_{j \geq 0} \beta^j \left[ \frac{1}{1 - \sigma_C} (C_{t+j}^h - h C_{t+j-1}^h)^{1 - \sigma_C} - \frac{\varepsilon_{t+j}^L \tilde{L}}{1 + \sigma_L} (L_{t+j}^h)^{1 + \sigma_L} \right] \varepsilon_{t+j}^B \right\}$$

Households obtain utility from consumption of a distribution good  $C_t^h$  (which also serves as an investment good), relative to an internal habit depending on past consumption, while receiving disutility from

its labour services  $L_t^h$ . Utility also incorporates a consumption preference shock  $\varepsilon_t^B$  and a labor supply shock  $\varepsilon_t^L$ .  $\tilde{L}$  is a positive scale parameter.

Financial markets are incomplete internationally. As assumed generally in the literature, *H*ome households can trade two nominal risk-less bonds denominated in the domestic and foreign currency. A risk premium as a function of real holdings of the foreign assets in the entire economy, is introduced on international financing of *H*ome consumption expenditures.

### 2.1.1 Intertemporal consumption plans

Each household  $h$  maximizes its utility function under the following budgetary constraint:

$$\frac{B_{H,t}^h}{\underline{P}_t R_t} + \frac{S_t B_{F,t}^h}{P_t R_t^* \varepsilon_t^{\Delta S} \Psi\left(\frac{\mathbb{E}_t S_{t+1}}{S_{t-1}} - 1, \frac{S_t (B_{F,t} - B_F^*)}{\underline{P}_t}\right)} + \frac{P_t C_t^h + I_t^h}{\underline{P}_t} =$$

$$\frac{B_{H,t-1}^h}{\underline{P}_t} + \frac{S_t B_{F,t-1}^h}{\underline{P}_t} + \frac{(1 - \tau_w) W_t^h L_t^h + A_t^h + TT_t^h}{\underline{P}_t} + R_t^k u_t^h K_t^h - \Phi(u_t^h) K_t^h + \frac{\Pi_t^h}{\underline{P}_t}$$

where  $W_t^h$  is the wage,  $A_t^h$  is a stream of income coming from state contingent securities,  $S_t$  is the nominal exchange rate,  $TT_t^h$  and  $\tau_{W,t}$  are government transfers and time-varying labor tax respectively, and

$$R_t^k u_t^h K_t^h - \Phi(u_t^h) K_t^h$$

represents the real return on the real capital stock minus the cost associated with variations in the degree of capital utilization. The income from renting out capital services depends on the level of capital augmented for its utilization rate and the cost of capacity utilization is zero when capacity are fully used ( $\Phi(1) = 0$ ).  $\Pi_t^h$  are the dividends emanating from monopolistically competitive intermediate firms. Finally,  $B_{H,t}^h$  and  $B_{F,t}^h$  are the individuals holding of domestic and foreign bonds denominated in local currency. The risk premium function  $\Psi(\bullet, \bullet)$  is differentiable, decreasing in both arguments and verifies  $\Psi(0, 0) = 1$ . Here, like [Adolfson et al. \(2007\)](#), we expanded the usual specification of the risk premium found in the open economy literature by introducing a term depending on the expected change in the exchange rate. As shown for example in the empirical work of [Duarte and Stockman \(2005\)](#), the forward risk premium on exchange rate is strongly negatively correlated with the expected depreciation. We also introduced a specific consumption tax which affect the price of the distributed goods serving final con-

sumption (and not investment). The after-tax consumer price index (CPI) is denoted  $P_t = (1 + \tau_{C,t}) \underline{P}_t$  where  $\underline{P}_t$  is the price of the distribution good gross of consumption tax. Such time-varying consumption tax could in principle rationalize the CPI inflation rate shocks that we introduce to estimate the model. We design the CPI shocks as  $\frac{(1+\tau_{C,t})}{(1+\tau_{C,t-1})} = \varepsilon_t^{CPI}$ . Thereafter, the functional forms used for the risk premium and for the adjustment costs on capacity utilization are given by  $\Psi(X, Y) = \exp(-\chi_{\Delta S} X - 2\chi Y)$  and  $\Phi(X) = \frac{R^{k*}}{\varphi} (\exp[\varphi(X-1)] - 1)$  for country  $H$  and  $\Phi(X) = \frac{R^{k*}}{\varphi^*} (\exp[\varphi^*(X-1)] - 1)$  for country  $F$ . Separability of preferences and complete financial markets domestically ensure that households have identical consumption plans.

### 2.1.2 Investment decisions

The capital is owned by households and rented out to the intermediate firms at a rental rate  $R_t^k$ . Households choose the capital stock, investment and the capacity utilization rate in order to maximize their intertemporal utility subject to the intertemporal budget constraint and the capital accumulation equation:

$$K_t = (1 - \delta)K_{t-1} + \varepsilon_t^I \left[ 1 - S\left(\frac{I_t}{I_{t-1}}\right) \right] I_t$$

where  $\delta \in (0, 1)$  is the depreciation rate,  $S$  is a non negative adjustment cost function such that  $S(1) = 0$  and  $\varepsilon_t^I$  is an efficiency shock on the technology of capital accumulation. The functional form used thereafter is  $S(x) = \frac{\phi}{2}(x-1)^2$  for country  $H$  and  $S(x) = \frac{\phi^*}{2}(x-1)^2$  for country  $F$ .

### 2.1.3 Wage-setting and labor supply

In country  $H$ , each household is a monopoly supplier of a differentiated labor service. For the sake of simplicity, we assume that he sells his services to a perfectly competitive firm which transforms it into an aggregate labor input using a CES technology  $L_t = \left[ \int_0^1 L_t(h)^{\frac{1}{\mu_w}} dh \right]^{\mu_w}$ , where  $\mu_w = \frac{\theta_w}{\theta_w - 1}$  and  $\theta_w > 1$  is the elasticity of substitution between differentiated labor services. The household faces a labor demand curve with constant elasticity of substitution  $L_t(h) = \left( \frac{W_t(h)}{W_t} \right)^{-\frac{\mu_w}{\theta_w - 1}} L_t$ , where  $W_t = \left( \int_0^1 W_t(h)^{\frac{1}{1-\mu_w}} dh \right)^{1-\mu_w}$  is the aggregate wage rate.

Households set their wage on a staggered basis. Each period, any household faces a constant probability  $1 - \alpha_w$  of optimally adjusting its nominal wage, say  $\widetilde{W}_t(h)$ , which will be the same for all sup-



pliers of labor services. Otherwise, wages are indexed on past inflation and steady state inflation:  $W_t(h) = [\Pi_{t-1}]^{\xi_w} [\Pi^*]^{1-\xi_w} W_{t-1}(h)$  with  $\Pi_t = \frac{P_t}{P_{t-1}}$ . Taking into account that they might not be able to choose their nominal wage optimally in a near future,  $\widetilde{W}_t(h)$  is chosen to maximize the intertemporal utility under the budget constraint and the labor demand for wage setters unable to re-optimize after period  $t$ .

## 2.2 Firms decision problems

### 2.2.1 Distribution goods

A continuum of companies operating under perfect competition mixes local production with imports. There is a home bias in the aggregation, which pins down the degree of openness at steady state. The distributor technology,  $\forall i \in [0, 1]$ , is given by:

$$Y_i = \left[ n_t^{\frac{1}{\xi}} Y_{i,H}^{\frac{\xi-1}{\xi}} + (1-n_t)^{\frac{1}{\xi}} Y_{i,F}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}$$

$$Y_i^* = \left[ (1-n_t^*)^{\frac{1}{\xi}} Y_{i,H}^*{}^{\frac{\xi-1}{\xi}} + n_t^*{}^{\frac{1}{\xi}} Y_{i,F}^*{}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}}$$

where  $\xi$  is the elasticity of substitution between bundles  $Y_H$  and  $Y_F$ . The degrees of home bias are subject to shocks. But as only the difference of openness rates enters the linearized aggregate equations in absence of adjustment costs on imports, home bias shocks are given by  $n_t = n\sqrt{\varepsilon_t^{\Delta n}}$  and  $n_t^* = \frac{n}{\sqrt{\varepsilon_t^{\Delta n}}}$ .

Before-tax distribution prices are defined by:

$$\underline{P}_t = P_{H,t} \left[ n_t + (1-n_t)T_t^{1-\xi} \right]^{\frac{1}{1-\xi}}$$

$$\underline{P}_t^* = P_{F,t}^* \left[ n_t^* + (1-n_t^*)T_t^{*\xi-1} \right]^{\frac{1}{1-\xi}}$$

where  $T = \frac{P_F}{P_H}$  and  $T^* = \frac{P_F^*}{P_H^*}$  denote the interior terms of trade. We also make use of the relative prices  $T_H = \frac{P_H}{\underline{P}}$  and  $T_F^* = \frac{P_F^*}{\underline{P}^*}$ .

### 2.2.2 Final goods

Final producers for local sales and imports are in perfect competition and aggregate a continuum of differentiated intermediate products from home and foreign intermediate sector.  $Y_H$  and  $Y_F$  are sub-

indexes of the continuum of differentiated goods produced respectively in country  $H$  and  $F$ . The elementary differentiated goods are imperfect substitutes with elasticity of substitution denoted  $\frac{\mu}{\mu-1}$ . Final goods are produced with the following technology  $Y_H = \left[ \int_0^1 Y(h)^{\frac{1}{\mu}} dh \right]^\mu$  and  $Y_F = \left[ \int_0^1 Y(f)^{\frac{1}{\mu}} df \right]^\mu$ . In the country  $F$ , the corresponding indexes are given by  $Y_F^* = \left[ \int_0^1 Y(f)^{\frac{1}{\mu}} df \right]^\mu$  and  $Y_H^* = \left[ \int_0^1 Y(h)^{\frac{1}{\mu}} dh \right]^\mu$ . For a domestic product  $h$ , we denote  $p(h)$  its price on local market and  $p^*(h)$  its price on the foreign import market. The domestic-demand-based price indexes associated with imports and local markets in both countries are defined as  $P_H = \left[ \int_0^1 p(h)^{\frac{1}{1-\mu}} dh \right]^{1-\mu}$ ,  $P_H^* = \left[ \int_0^1 p^*(h)^{\frac{1}{1-\mu}} dh \right]^{1-\mu}$ ,  $P_F^* = \left[ \int_0^1 p^*(f)^{\frac{1}{1-\mu}} df \right]^{1-\mu}$  and  $P_F = \left[ \int_0^1 p(f)^{\frac{1}{1-\mu}} df \right]^{1-\mu}$ .

### 2.2.3 Intermediate goods

Intermediate goods are produced with a Cobb-Douglas technology as follows:

$$\begin{cases} \forall h \in [0, 1], & Y_t(h) = \varepsilon_t^A (u_t K_{t-1}(h))^\alpha L_t(h)^{1-\alpha} - \Omega \\ \forall f \in [0, 1], & Y_t^*(f) = \varepsilon_t^{A^*} (u_t^* K_{t-1}^*(f))^\alpha L_t^*(f)^{1-\alpha} - \Omega \end{cases}$$

where  $\varepsilon_t^A$  and  $\varepsilon_t^{A^*}$  are exogenous technology parameters. Each firm sells its products in the local market and in the foreign market. We denote  $Y_H(h)$  and  $Y_H^*(h)$  (respectively  $Y_F^*(f)$  and  $Y_F(f)$ ) the local and foreign sales of domestic producer  $h$  (respectively foreign producer  $f$ ) and we define  $L_H(h)$  and  $L_H^*(h)$  (respectively  $L_F^*(f)$  and  $L_F(f)$ ) the corresponding labor demand.

Firms are monopolistic competitors and produce differentiated products. For local sales, firms set prices on a staggered basis *à la* Calvo. In each period, a firm  $h$  (resp.  $f$ ) faces a constant probability  $1 - \alpha_H$  (resp.  $1 - \alpha_F^*$ ) of being able to re-optimize its nominal price. This probability is independent across firms and time in a same country. The average duration of a rigidity period is  $\frac{1}{1-\alpha_H}$  (resp.  $\frac{1}{1-\alpha_F^*}$ ). If a firm cannot re-optimize its price, the price evolves according to the following simple rule:

$$p_t(h) = \Pi_{H,t-1}^{\gamma_H} \Pi^{\star 1-\gamma_H} p_{t-1}(h)$$

Otherwise, firm  $h$  chooses  $\hat{p}_t(h)$  to maximize its intertemporal profit. We introduce a time varying tax on firms' revenue which is affected by an i.i.d shock defined by  $1 - \tau_t = (1 - \tau^*) \varepsilon_t^P$ .

Concerning exports, we assume that, in country  $H$ , a fraction  $\eta$  (respectively  $\eta^*$  in country  $F$ ) of exporters exhibit producer-currency-pricing (PCP) while the remaining firms exhibit local-currency-pricing

(LCP). Consequently, aggregate export prices denominated in foreign currency are given by:

$$P_H^* = \left[ \eta \left( \frac{P_{H,t}}{S_t} \right)^{\frac{1}{1-\mu}} + (1-\eta) \tilde{P}_H^{*\frac{1}{1-\mu}} \right]^{1-\mu}, \text{ and } P_F = \left[ \eta^* (S_t P_{F,t}^*)^{\frac{1}{1-\mu}} + (1-\eta^*) \tilde{P}_F^{\frac{1}{1-\mu}} \right]^{1-\mu}.$$

The aggregate LCP export price indexes are accordingly defined as

$$\tilde{P}_H^* = \left[ \frac{1}{1-\eta} \int_{\eta}^1 p^*(h)^{\frac{1}{1-\mu}} dh \right]^{1-\mu}, \text{ and } \tilde{P}_F = \left[ \frac{1}{1-\eta^*} \int_{\eta^*}^1 p(f)^{\frac{1}{1-\mu}} df \right]^{1-\mu}.$$

Let us define the following relative prices  $R\tilde{E}R_H = \frac{S\tilde{P}_H^*}{P_H}$ ,  $R\tilde{E}R_F = \frac{\tilde{P}_F}{S\tilde{P}_F^*}$  and  $\tilde{T} = \frac{\tilde{P}_F}{P_H}$ . Export margins relative to local sales are denoted  $RE R_H = \frac{SP_H^*}{P_H}$  and  $RE R_F = \frac{P_F}{S\tilde{P}_F^*}$ . If there is some form of international price discrimination, those ratios figure the relative profitability of foreign sales compared with the local ones. LCP exporters also set their prices on a staggered basis and features of nominal rigidities are the same as for the local producers.

## 2.3 Government

In country  $H$ , public expenditures  $G^*$  are subject to random shocks  $\varepsilon_t^G$ . The government finances public spending with the various taxes and lump-sum transfers.

The government also controls the short term interest rate  $R_t$ . Monetary policy is specified in terms of an interest rate rule: the monetary authority follows generalized Taylor rules which incorporate deviations of lagged inflation and the lagged output gap defined as the difference between actual and flexible-price output. Such reaction functions also incorporate a non-systematic component  $\varepsilon_t^R$ . We assumed that monetary authorities target domestic objectives: the domestic detrended output and CPI inflation rate. Written in deviation from the steady state, the interest feedback rule used in the estimation has the form:

$$r_t = \rho r_{t-1} + (1-\rho) [r_\pi \pi_{t-1} + r_y z_{t-1}] + r_{\Delta\pi} \Delta\pi_t + r_{\Delta y} \Delta z_t + \log(\varepsilon_t^R)$$

where small case variables denote log-deviation from its deterministic steady-state.

## 3 Estimation

In this section, we describe the DSGE-VAR methodology and present the estimation on a US and euro area dataset of the first order approximation of the model described in the previous section. We closely

follow the econometric approach used by [Del Negro et al. \(2006\)](#) who estimated a medium-scale closed-economy model on US data. Regarding the open economy literature, various studies have attempted to bring multi-country models on data over the recent years. More specifically, we could refer to the results of [De Walque et al. \(2005\)](#), [Rabanal and Tuesta \(2006\)](#), [Bergin \(2006\)](#) or [Adolfson et al. \(2005\)](#). However, up to our knowledge, fewer studies are available concerning multi-country DSGE-VAR estimation (see for example [Adolfson et al. \(2007\)](#)).

Thereafter, country  $H$  represents the US and country  $F$  the euro area. Concerning the structural shocks introduced in the estimation, we chose to keep a large set of domestic shocks as in [Smets and Wouters \(2005\)](#). The exogenous can be divided in three categories:

1. Efficient shocks: AR(1) shocks on technology  $(\epsilon_t^A, \epsilon_t^{A*})$ , investment  $(\epsilon_t^I, \epsilon_t^{I*})$ , labor supply  $(\epsilon_t^L, \epsilon_t^{L*})$ , public expenditures  $(\epsilon_t^G, \epsilon_t^{G*})$ , consumption preferences  $(\epsilon_t^B, \epsilon_t^{B*})$  and relative home bias  $\epsilon_t^{\Delta n}$ .
2. Inefficient shocks: i.i.d. shocks on PPI markups  $(\epsilon_t^P, \epsilon_t^{P*})$ , CPI markups  $(\epsilon_t^{CPI}, \epsilon_t^{CPI*})$ , and UIP  $(\epsilon_t^{\Delta S})$ .
3. Policy shocks: shocks on short term interest rates  $(\epsilon_t^R, \epsilon_t^{R*})$ .

### 3.1 Data

For each country we potentially consider 8 key macro-economic quarterly time series from 1972q1 to 2005q4: output, consumption, investment, hours worked, real wages, GDP deflator inflation rate, CPI inflation rate and 3 month short-term interest rate. US series come from BEA and BLS. Euro area data are taken from Fagan et al (2001) and Eurostat. Concerning the euro area, employment numbers replace hours. Consequently, as in [Smets and Wouters \(2005\)](#), hours are linked to the number of people employed  $e_t^*$  with the following dynamics:

$$e_t^* = \beta \mathbb{E}_t e_{t+1}^* + \frac{(1 - \beta \lambda_e)(1 - \lambda_e)}{\lambda_e} (l_t^* - e_t^*)$$

The exchange rate is the euro/dollar exchange rate. Due to statistical problems in computing long series of bilateral current account and current account for the euro area, we used the US current account as a share of US GDP. Aggregate real variables are expressed per capita by dividing with working age population.

All the data are detrended before the estimation. Such pre-filtering is somewhat unnatural within the econometric literature using vector autoregressions whereas many estimated DSGE models rely on some filtering methods to match stationary data or impose some structural co-integration relationships which, most of the time, are rejected empirically. Here, for comparison purposes, we wanted to use the same dataset and data transformation as in [Adjemian et al. \(2007\)](#). Therefore, the estimation results provided in this paper are strictly comparable to the ones reported in their study. In addition, the main focus of this paper is on the international propagation mechanism of shocks and we do not investigate carefully the in and out-of-sample empirical properties of the DSGE-VAR which may be more dependent on the use of raw data instead of linearly detrended ones.

Actually, compared with the estimation of [Adjemian et al. \(2007\)](#), we limited the number of shocks to be equal to the number of observed variables. Therefore, no common shocks were introduced while the wage markup shocks and external risk premium shocks have been omitted. Differently from our previous estimation, we allowed for correlation between preference shocks and external risk premium shocks essentially to match the correlation between consumption and investment present in the data.

We also introduced a correlation between the home bias preference shock and the euro area public expenditure shock. Since we used the US total net trade instead of the bilateral net trade, we intend to capture through this variable shocks that affect the US current account with moderate immediate impact on euro area output. The correlation between home bias shock and euro area public expenditures shock ( $\rho_{\Delta n, G}$ ), which acts as a GDP residual shock, is meant to control for this drawback. Notice however that using total US trade instead of bilateral trade broadens the data information on the rest of the world.

Finally, given that, in the first order approximation of the model, the UIP shock has weak structural interpretation, examining the links with other shocks seems justified. Consequently, correlations between the UIP shock and other efficient shocks are incorporated in the estimation and may account for the impact of fundamental shocks on time-varying risk premium. In practice, the benchmark model exposed in this section features a correlation between the UIP shocks and the US productivity shocks ( $\rho_{A, \Delta S}$ ) as well as the government expenditure shocks ( $\rho_{G, \Delta S}$ ,  $\rho_{G^*, \Delta S}$ ) from both countries.

## 3.2 The DSGE-VAR approach

A common practice for assessing the empirical coherence of DSGEs is to compare its marginal density to those of more general and less economically constrained models like BVAR models. This comparison of course suffers from several limitations. First, bayesian model probabilities are obviously sensitive to the choice of priors and more specifically, some clear guidance is missing on the appropriate choice of priors for BVARs. Second, an unrestricted finite order VAR model is not really more general than a DSGE. Indeed, a DSGE model generates an infinite order vector moving average of which a finite order VAR is only an approximation. Finally, such comparison of marginal densities is uninformative about the directions in which the DSGE model is more or less successful.

[Del Negro and Schorfheide \(2004\)](#) answers the first and the last limitations by building the priors of a BVAR model from a DSGE model and evaluate the optimal weight of the DSGE priors. Their approach still relies on a finite order VAR representation of the DSGE but the error of approximation should be relatively minor, at least with a reasonably larger lag length in the VAR. This approach is relatively easy to implement. The shape of the priors is carefully chosen while using a mixed estimation technique: the posterior density is obtained from the likelihood function by augmenting the sample with artificial data generated by the DSGE model. The size of the artificial sample,  $\mathcal{T}$  relative to the data sample  $T$ , defines the weight of the prior information relative to the likelihood. Let us denote  $\lambda = \frac{\mathcal{T}}{T}$ . A crucial issue is to choose the “optimal” weight,  $\lambda$ , of the DSGE prior in the BVAR model. An “optimal” high value of  $\lambda$  means that the DSGE model imposes useful restrictions to improve the (in sample) predictive properties of the BVAR model. Conversely, a low value of  $\lambda$  indicates that a minimal use of the DSGE restrictions on the priors of the BVAR is preferred, therefore casting doubts on the coherence of the DSGE model with the data.

### 3.2.1 Deriving the posterior densities

Consider the order  $p$  VAR representation for the  $1 \times m$  vector of observed variables  $y_t$ :

$$y_t = \sum_{k=1}^p y_{t-k} \mathbf{A}_k + u_t$$

where  $u_t \sim \mathcal{N}(0, \Sigma_u)$ . Let  $z_t$  be the  $mp \times 1$  vector  $[y'_{t-1}, \dots, y'_{t-p}]'$  and define  $\mathbf{A} = [\mathbf{A}'_1, \dots, \mathbf{A}'_p]'$ , the VAR representation can then be written in matrix form as:

$$Y = Z\mathbf{A} + \mathcal{U}$$

where  $Y = (y'_1, \dots, y'_T)'$ ,  $Z = (z'_1, \dots, z'_T)'$  and  $\mathcal{U} = (u'_1, \dots, u'_T)'$ .

Dummy observations prior for the VAR can be constructed using the VAR likelihood function for  $\mathcal{T} = [\lambda T]$  artificial data simulated with the DSGE  $(Y^*, Z^*)$ , combined with diffuse priors. The prior is then given by:

$$p_0(\mathbf{A}, \Sigma \mid Y^*, Z^*) \propto |\Sigma|^{-\frac{\lambda T + m + 1}{2}} e^{-\frac{1}{2} \text{tr}[\Sigma^{-1}(Y^{*'}Y^* - \mathbf{A}'Z^{*'}Y^* - Y^{*'}Z^*\mathbf{A} + \mathbf{A}'Z^{*'}Z^*\mathbf{A})]}$$

implying that  $\Sigma$  follows an inverted Wishart distribution and  $\mathbf{A}$  conditional on  $\Sigma$  is gaussian. Assuming that observables are covariance stationary, [Del Negro and Schorfheide \(2004\)](#) use the DSGE theoretical autocovariance matrices for a given  $n \times 1$  vector of model parameters  $\theta$ , denoted  $\Gamma_{YY}(\theta)$ ,  $\Gamma_{ZY}(\theta)$ ,  $\Gamma_{YZ}(\theta)$ ,  $\Gamma_{ZZ}(\theta)$  instead of the (artificial) sample moments  $Y^{*'}Y^*$ ,  $Z^{*'}Y^*$ ,  $Y^{*'}Z^*$ ,  $Z^{*'}Z^*$ . In addition, the  $p$ -th order VAR approximation of the DSGE provides the first moment of the prior distributions through the population least-square regression:

$$\mathbf{A}^*(\theta) = \Gamma_{ZZ}(\theta)^{-1} \Gamma_{ZY}(\theta) \tag{P1a}$$

$$\Sigma^*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YZ}(\theta)\Gamma_{ZZ}(\theta)^{-1} \Gamma_{ZY}(\theta) \tag{P1b}$$

Conditional on the deep parameters of the DSGE  $\theta$  and  $\lambda$ , the priors for the VAR parameters are given by:

$$\text{vec}\mathbf{A} \mid \Sigma, \theta, \lambda \sim \mathcal{N}\left(\text{vec}\mathbf{A}^*(\theta), \Sigma \otimes [\lambda T \Gamma_{ZZ}(\theta)]^{-1}\right) \tag{P2}$$

$$\Sigma \mid \theta, \lambda \sim \mathcal{IW}(\lambda T \Sigma^*(\theta), \lambda T - mp - m)$$

where  $\Gamma_{ZZ}(\theta)$  is assumed to be non singular and  $\lambda \geq \frac{mp+m}{T}$  for the priors to be proper<sup>1</sup>. The *a priori* density of  $\mathbf{A}$  is defined by  $n + 1$  parameters ( $\theta$  and  $\lambda$ ), which is likely to be less than  $mp$  (the VAR number of parameters). If we have a one-to-one relationship (no identification issues) between  $(\theta, \lambda)$  and  $\mathbf{A}$  it will be a good idea to estimate  $(\theta, \lambda)$  instead of  $\mathbf{A}$ , *ie* to estimate fewer free parameters. To

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<sup>1</sup>Note that it would not be possible to estimate the VAR model by OLS (or maximum likelihood) if we had  $T < m(p + 1)$ . In this case we would not have more observations than parameters to estimate.

do so, [Del Negro and Schorfheide \(2004\)](#) complete the prior by specifying a prior distribution over the structural model's deep parameters:  $p_0(\theta)$ . We still have to set the weight of the structural prior,  $\lambda$ . [Del Negro and Schorfheide](#) choose the value of  $\lambda$  that maximizes the marginal density. They estimate a limited number of DSGE-VAR models with different values of  $\lambda$ . For each model they also estimate the marginal density and select the model (*ie* the value of  $\lambda$ ) with highest marginal density. In the present paper, we estimate directly  $\lambda$  as another parameter, instead of doing a loop over the values of this parameter<sup>2</sup>. So we define a prior on the distribution of  $\lambda$ , which is assumed to be independent from  $\theta$ . Finally, the DSGE-VAR model has the following prior structure:

$$p(\mathbf{A}, \Sigma, \theta, \lambda) = p_0(\mathbf{A}, \Sigma | \theta, \lambda) \times p_0(\theta) \times p_0(\lambda) \quad (\text{P3})$$

where  $p_0(\mathbf{A}, \Sigma | \theta, \lambda)$  is defined by [\[P1a,P1b\]](#) and [\[P2\]](#).

The posterior distribution, may be factorized in the following way:

$$p(\mathbf{A}, \Sigma, \theta, \lambda | \mathcal{Y}_T) = p(\mathbf{A}, \Sigma | \mathcal{Y}_T, \theta, \lambda) \times p(\theta, \lambda | \mathcal{Y}_T) \quad (\text{Q3})$$

where  $\mathcal{Y}_T$  stands for the sample. A closed form expression for the first density function on the right hand side of [\[Q3\]](#) is available. Conditional on  $\theta$  and  $\lambda$ , [\[P1a,P1b\]](#) and [\[P2\]](#) define a conjugate prior for the VAR model, so its posterior density has to belong to the same family: the distribution of  $\mathbf{A}$  conditional on  $\Sigma, \theta, \lambda$  and the sample is matrix-variate normal, and the distribution of  $\Sigma$  conditional on  $\theta, \lambda$  and the sample is inverted Wishart. More formally, we have:

$$\begin{aligned} \text{vec} \mathbf{A} | \Sigma, \theta, \lambda, \mathcal{Y}_T &\sim \mathcal{N} \left( \text{vec} \tilde{\mathbf{A}}(\theta, \lambda), \Sigma \otimes V(\theta, \lambda)^{-1} \right) \\ \Sigma | \theta, \lambda, \mathcal{Y}_T &\sim \text{IW} \left( (\lambda + 1)T \tilde{\Sigma}(\theta, \lambda), (\lambda + 1)T - mp - m \right) \end{aligned} \quad (\text{Q2})$$

where:

$$\tilde{\mathbf{A}}(\theta, \lambda) = V(\theta, \lambda)^{-1} (\lambda T \Gamma_{ZY}(\theta) + Z'Y) \quad (\text{Q1a})$$

$$\tilde{\Sigma}(\theta, \lambda) = \frac{1}{(1 + \lambda)T} [\lambda T \Gamma_{YY}(\theta) + Y'Y - (\lambda T \Gamma_{YZ}(\theta) + Y'Z) V(\theta, \lambda)^{-1} (\lambda T \Gamma_{ZY}(\theta) + Z'Y)] \quad (\text{Q1b})$$

with:

$$V(\theta, \lambda) = \lambda T \Gamma_{ZZ}(\theta) + Z'Z$$

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<sup>2</sup>In this regard, the approach followed by [Del Negro and Schorfheide](#) is, at least computationally, inefficient. Also, contrary to us, they do not average over different possible values of  $\lambda$  but pick a single value of this parameter, which is not the bayesian way.



Not surprisingly, we find that the posterior mean of  $\mathbf{A}$  is a convex combination of  $A^*(\theta)$ , the prior mean, and of the OLS estimate of  $\mathbf{A}$ . When  $\lambda$  goes to infinity the posterior mean shrinks towards the prior mean, *ie* the projection of the DSGE model onto the VAR( $p$ ).

We do not have a closed form expression for the joint posterior density of  $\theta$  and  $\lambda$  (the second term on the right hand side of [Q3]). So the posterior distribution of  $(\theta, \lambda)$  is recovered from an MCMC algorithm, as described in (Del Negro and Schorfheide 2004; appendix B), except that we do estimate  $\lambda$  as the deep parameters  $\theta$ .<sup>3</sup>

All in all, this estimation procedure allows to select the “best specification ” from a continuum of intermediate models indexed by  $\lambda$  and ranging between the VAR( $p$ ) with diffuse priors and the VAR( $p$ ) approximation of the DSGE model. The posterior distribution of the deep parameters,  $\theta$ , can then be interpreted as the best model to be used as a prior for the corresponding VAR( $p$ ). In addition, the posterior distribution of  $\lambda$  gives an indication of the reliability of the DSGE model and of the empirical relevance of the associated economic restrictions.

Notice that, when  $\lambda$  is closer to its lowest possible value, the DSGE-VAR approximates an unrestricted VAR with diffuse priors. Given the number of observed variables in our study, such VAR has obviously poor empirical performance due to many free parameters and associated sampling errors. The econometric literature has shown that Bayesian VARs can improve on unrestricted VAR by introducing “Minnesota-like ” priors for example, which favor persistence, low cross-variable interactions and smaller coefficients at distant lags. Therefore, the DSGE-VAR approach is bound to call for higher  $\lambda$  to increase the tightness of priors regarding serial correlations for instance which does not necessarily mean that the economic restrictions of the DSGE are more consistent with the data. A way to circumvent this problem would be to introduce an other source of dummy observation coming from a BVAR with some version of Minnesota priors and let the procedure infer the relative weight to put to the DSGE priors and the “Minnesota priors ”. This avenue is left for further research.

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<sup>3</sup>This can be done with [Dynare 4](#).

### 3.2.2 Identification

In [Del Negro and Schorfheide \(2004\)](#) the DSGE-VAR approach is shown to provide a quite natural identification scheme for the structural innovations. In the sequel we follow the aforementioned authors. The sole difference is, again related to  $\lambda$ . Our Impulse Response Functions are obtained by averaging over the posterior distribution of  $\lambda$ .

### 3.3 Prior distribution of parameters

Some parameters are fixed prior to estimation. This concerns generally parameters driving the steady state values of the state variables for which the econometric model including detrended data is quasi uninformative. Those parameters are assumed to be the same for the US and the euro area. The discount factor  $\beta$  is calibrated to 0.99, which implies annual steady state real interest rates of 4%. The depreciation rate  $\delta$  is equal to 0.0025 per quarter. Markups are 1.3 in the goods market and 1.5 in the labor market. The steady state is consistent with labor income share in total output of 60%. Shares of consumption and investment in total output are respectively 0.65 and 0.18.

As in [Smets and Wouters \(2005\)](#), the priors are assumed to be the same across countries. The standard errors of the innovations are assumed to follow uniform distributions. In DSGE models, data are often very informative about the variance of structural disturbances and we keep loose priors to avoid helping artificially the identification of our shock structure by our assumptions on priors. The distribution of the persistence parameters in the efficient and policy shocks is assumed to follow a beta distribution with mean 0.85 and standard error 0.1. Concerning the parameters of the Taylor rules, we follow [Smets and Wouters \(2005\)](#): the long run coefficient on inflation and output gap are described by a Normal distribution with mean 1.5 and 0.125, and standard errors 0.1 and 0.05 respectively. The persistence parameter follows a normal around 0.75 with a standard error of 0.1. The prior on the short run reaction coefficients to inflation and output gap changes reflect the assumptions of a gradual adjustment towards the long run. Concerning preference parameters, the intertemporal elasticity of substitution is set at 1 with standard error of 0.375. The habit parameter is centered on 0.7 with standard deviation of 0.1 and the elasticity of labor supply has mean 2 and standard error of 0.75. Adjustment cost parameter for investment follows a  $\mathcal{N}(4, 2)$  and the capacity utilization elasticity is set at 0.2 with a standard error of 0.1. Concerning the Calvo probabilities of price and wage settings, we assume a beta distribution around

0.75. The degree of indexation to past inflation is centered on 0.5.

Regarding the open economy parameters, we intend to remain fully agnostic on such parameters and choose uniform priors for the intratemporal elasticity of substitution, the parameters guiding the share of PCP producers, the degree of home bias in consumption and the elasticity of foreign exchange risk premium with respect to the net foreign assets.

Note therefore that the steady state value of the openness ratio is estimated. As a rest of the world bloc is not included in our framework, we try to “let the data speak” about the effective openness ratio in this reduced form model of the international linkages between the US and the euro area. One interesting point is to see if the estimated openness ratio is closer to the bilateral openness, around 2%, than to the overall openness, above 13%.

Finally we set an uniform prior for  $\lambda$ . Given that this parameter would go to infinity if the DSGE was the true data generating process, we also made sure that the MCMC algorithm used to recover the posterior distribution could explore a large space on this dimension. Therefore, the prior distribution was first set on the interval  $[0,100]$  and then on the interval  $[0,5]$ . The estimation results were quasi identical with both priors (not reported here), then we chose the smallest interval not to deteriorate artificially the marginal data density of the model by integrating on low likelihood parameter space.

### 3.4 Posterior parameter estimates

The estimation results reported in this section have been performed with a DSGE-VAR of order 4. We first proceed by computing the log marginal density for the DSGE-VAR. Compared with [Del Negro et al. \(2006\)](#) or [Del Negro and Schorfheide \(2007\)](#), we estimate in our procedure the share of dummy observation  $\lambda$  together with all the structural parameters while in those studies, a discrete grid on  $\lambda$  is performed and the value with the highest log marginal data density is retained. The posterior mode for  $\lambda$  is around 2.2 with a relatively close high density interval ranging from 1.9 to 2.5. This value is much higher than the one obtained in [Del Negro et al. \(2006\)](#) or [Del Negro and Schorfheide \(2007\)](#) with values between 0.75 and 1. This would tend to suggest that the cross-country restrictions on the VAR

representation imposed by our model relatively more useful than the closed-economy ones. Plausibly, the fact that the two-country DSGE induces very limited cross-country interactions may help reducing the dimension of autoregressive parameters in the VAR form with  $18 \times 18 \times 4$  dimension: imagine that the US-EA had no linkages and no common shocks, then the 18 variables VAR we use would be highly over parameterized and a prior imposing zero cross-county interactions would strongly improve the inference. At the same time, one should be cautious in comparing estimated values for  $\lambda$  across different models and dataset. Recall indeed that in the DSGE-VAR methodology, the priors are only well-defined for  $\lambda \geq \frac{mp+m}{T}$ . In our case, this critical value is around 0.6 while for the two other studies it averages 0.3. The log marginal data density for the DSGE-VAR is then -1751.7 against -1881.5 for the direct DSGE estimation and -1824.2 for the "augmented" model similar to [Adjemian et al. \(2007\)](#) specification, adding correlations between preference and external risk premium shocks. The introduction of further shocks and sources of cross-country correlation and persistence can also be seen as a first attempt to account for misspecifications in the theoretical foundations of the model and indeed, the marginal data density is significantly higher. But overall, the considerable advantage for the DSGE-VAR in terms of marginal data density indicates that deviations from the cross-equation restrictions associated with the theoretical model are needed the data and that the generalization of the shock structure is not sufficient.

In the following, we turn to parameter estimates and systematically compare the DSGE-VAR coefficients (see Tables 1 to 3 and Figures 1 to 3) with one derived from the direct estimation of the DSGE described in section 2 and the "augmented" version (see Tables 4-6). As already mentioned, the differences between both models specifications come from the addition of country-shocks on wage markups and external risk premium as well as AR(1) common factors.

With this direct estimation, the DSGE is forced to match all observed fluctuations. In the DSGE-VAR estimation however, the parameters are somewhat determined to minimize the distant between the VAR coefficients and the cross-equation restrictions imposed by the VAR representation of the DSGE model. Therefore, the DSGE-VAR parameter estimates allow some of the economic fluctuations to be explained by deviations from the cross-equation restrictions. If those restrictions cannot be relaxed in the estimation, the misspecifications will then be absorbed by some structural parameters which will show up in the comparison between the direct and DSGE-VAR estimates.

First, regarding preferences parameters, the DSGE-VAR estimation presents a slightly lower intertemporal elasticity of substitution for the euro area than in the "augmented" version . The labor supply elasticity for the US is also lower compared with both direct estimations but those parameters are still badly identified. Concerning nominal rigidity parameters, the DSGE-VAR estimation delivers generally more limited nominal frictions than the direct estimation, in line with the results of [Del Negro et al. \(2006\)](#): more specifically, the Calvo wage-setting parameter for both countries, the Calvo parameter on price-setting for the euro area, the price and wage indexation for the US are significantly lower compared with the direct estimation. To a minor extent, the "augmented" version also presents lower degree of nominal rigidity. Similarly, the adjustment costs on investment and capacity utilization seem to be slightly lower in the DSGE-VAR estimation than in the direct estimation except for the euro area capacity utilization parameter.

Concerning monetary policy rules, there is not much evidence of strong differences in the estimation of the reaction functions except for the level terms on output, with higher coefficients for the DSGE-VAR estimation and for the difference term on inflation which is higher for the euro area and lower for the US. Note that the DSGE-VAR approach does not help better identifying the level terms on inflation in the policy rules also reported by [Adjemian et al. \(2007\)](#).

Turning now to the open economy parameters, the DSGE-VAR estimation delivers coefficients similar to the ones from the direct estimation. The price elasticity of trade ( $\xi$ ) is estimated around 2.1 in the benchmark model with the highest probability density interval going approximately from 1.1 to 3.3. Note however that, as in the direct estimation cases, such estimate is very sensitive to model specifications and to the variables we introduce in the estimation. For the configurations we investigated, this posterior distribution is one of the highest. Moreover, as in direct estimations, we estimate the share of PCP and LCP firms (given by the parameters  $\eta$  and  $\eta^*$ ) to be close to 0.9 for the US and 0.8 for the euro area. The DSGE-VAR estimation also features a degree of home bias consistent with the bilateral openness ratio between the US and the euro area. However, in the DSGE-VAR, the coefficient on the risk premium of the UIP linked to net foreign assets was not well-identified. Therefore we did not report the results with the uniform distribution but one performed with a prior gamma distribution which indeed

presents a posterior distribution quasi identical to the prior one.

Overall, the DSGE-VAR estimation implies behavioral parameters which are broad in line with the main features highlighted by [Adjemian et al. \(2007\)](#). Marginally, the observed differences show most of the time that the DSGE-VAR estimates lie between the prior and the direct DSGE estimation. Indeed, with low value for  $\lambda_{DSGE}$ , the marginal likelihood will become flatter in the direction of the structural parameters and the priors will therefore gain more weight in determining the posteriors.

However, the differences are much stronger on the stochastic properties of the structural disturbances. In this respect, the direct estimates and the ones from the "augmented" version differ sensibly due to the different specification of structural shocks. The comparison with the DSGE-VAR is therefore more meaningful if limited to the direct estimation. First, the persistence coefficients of most AR(1) shock processes are generally lower in the DSGE-VAR estimation. All the persistence parameters are significantly lower except for the home bias shock and the euro area public expenditure shock. Moreover, most of the standard deviations of structural shocks are reduced in the DSGE-VAR estimation. By allowing for model-misspecifications, the DSGE-VAR leads indeed to smaller forecast errors and ultimately smaller shock volatility estimates. Note that the augmented DSGE specification also generates highly serially correlated direct estimates. [Del Negro and Schorfheide \(2007\)](#) also report higher persistence and volatility of shock processes in the DSGE-VAR estimation. Consequently, the structural description of business cycle fluctuations provided by the DSGE model is strongly modified when accounting for model misspecifications. A shock decomposition of variance in the DSGE model using both sets of parameters (not reported here) would show that the contribution to efficient supply shocks to activity for example is much lower with the DSGE-VAR estimates while the role of demand and policy shocks increases.

Finally, the correlations we allowed between the structural shocks and the UIP shock were retained for US productivity shocks and for both US and EA government spending shocks. With the direct and DSGE-VAR estimation, the correlations amplify the reaction of the exchange rate: the exchange depreciates more after the productivity shock while the appreciation induced by the government expenditure shock is larger. Regarding the correlation between the home bias shock and the EA government spending shock which we introduced to control for the introduction of the total US current account instead of

the bilateral one, the posterior estimate comes out relatively high and well-identified. This will partially break the asymmetry of the propagation of the home bias shock which pushes the output in opposite directions in the two countries. The correlations between preference and equity risk premium shock are also very well pinned down by both estimation methods and allow to better capture comovements between consumption and investment.

## 4 Assessing US-EA interdependence

In this section, we focus on two dimensions of the DSGE-VAR methodology to explore the economic interdependence between the US and the euro area. First, the estimated DSGE-VAR is equivalent to a structurally-identified VAR and as such, provides evidence on the domestic and international transmission of a relatively large set of structural shocks. Second, this approach is meant to illustrate the potential misspecifications of the theoretical model: comparing the propagation of shocks in the DSGE-VAR and in the DSGE sheds some lights on the dimensions of the model which may not be well-supported by the data.

In the following, we systematically compare the properties of the estimated DSGE-VAR and the DSGE using the same set of parameters. We first look at the implied moments of both models and in particular cross-country correlations. Then, we analyze the structural decomposition of such moments as explained by the DSGE-VAR and the DSGE. Finally, the comparison of propagation mechanisms allows to complement and enrich some conclusions drawn from the decomposition of moments.

### 4.1 Matching selected moments

First, we compare selected moments implied by our DSGE-VAR estimation with those from the sample data and from the DSGE model using the parameter set from the DSGE-VAR estimation (see Table 7). Note that we reported in Table 7 data moments for different samples due to some instability with respect to the starting date.

Concerning standard deviations, the estimated DSGE-VAR implies, across the board, higher volatilities than the DSGE model and in particular for real variables. Compared to the data however, the DSGE-VAR induces generally lower variances. This contrasts with the results of [Adjemian et al. \(2007\)](#) which show that the direct estimation of the DSGE delivers significantly higher volatility of real variables than in the data. As we already mentioned, the direct estimation is likely to capture misspecifications through a mix of higher persistence and higher volatility of structural shocks which then tend to generate elevated theoretical moments compared with the data.

Turning to cross-country correlations, the DSGE model is not able to display the appropriate levels of correlations present in the data. The only moment which is relatively well accounted for is the correlation between the depreciation rate and the US current account. The DSGE direct estimation of [Adjemian et al. \(2007\)](#) was relatively more successful in matching those moments but relied on explicit common factors to explain co-movements. The DSGE-VAR is featuring more adequate cross-country correlations for output, real wages, inflation rates and interest rate. However, even if the conditional cross-correlation of consumption is positive at a below-three-year horizon, the unconditional moment is strongly negative which is at odds with the positive correlation present in the data for most of the samples reported here. For investment and labor, conditional cross-country correlations are positive at short horizons but decrease afterwards. In the data, the sample is strongly affecting the empirical correlations which tend to diminish with the most recent samples. It is therefore more difficult to assess the performance of the DSGE-VAR concerning those moments. Overall, it seems that the DSGE-VAR is providing sensible cross-correlations except for consumption.

In order to explain those results, we will first try to illustrate how the sources of economic fluctuations affect the variances and covariances described previously with the DSGE-VAR and the DSGE.

## 4.2 Structural sources of business cycle fluctuations

Table 8 presents the shock decomposition of unconditional variances for the DSGE-VAR and the DSGE while conditional shock decomposition for selected cross-country covariances are shown in Tables 9 -12. A broad feature of the shock decomposition shown in Table 8 is the sharp increase in foreign shocks



contributions to domestic variables in the DSGE-VAR compared with the DSGE. The role of foreign disturbances seems even more important for the US than for the euro area. 30% of US GDP fluctuations are due to the euro area in the DSGE-VAR against 1.5% in the DSGE. Similarly, 15% of euro area GDP are explained by the US compared with 1.8% in the DSGE. On inflation and interest rates, the euro area contributes to around 20% of US fluctuations while the US is driving about 10% of euro area variances in the DSGE-VAR. The corresponding numbers for the DSGE only averages 3%. Conditional variance decompositions (not reported here) also show that on activity and to a lesser extent on inflation and interest rate, the contribution of foreign shocks during the 2-to-10-year horizon increase sharply in the DSGE-VAR while it remains relatively flat in the DSGE. At this stage, it seems that the DSGE-VAR generates higher and more persistent spillovers in the medium term than the DSGE model. However, the stronger relative contribution of euro area shocks on the US economy as portrayed by the DSGE-VAR is somewhat surprising compared with results found in the econometric literature which generally emphasizes that spillovers, if present, run from the US to the other countries more than in the opposite direction (see for instance [Dées and Vansteenkiste 2007](#)).

Looking in more details at the individual shock contributions, the increased role of foreign shocks on activity in the DSGE-VAR should in principle imply an homogenous decrease in domestic shocks contributions. However, it appears that the contributions of domestic markups and policy shocks in particular even increase in the DSGE-VAR compared with the DSGE. In terms of spillovers on output, the international transmission of most structural shocks is increased in the DSGE-VAR. More specifically, policy shocks, labor supply, consumer preference and PPI-markup have significantly higher cross-country contributions in the DSGE-VAR than in the DSGE. Between the two models, the role of UIP and home bias shocks is relatively similar for the euro area. In the US, the home bias shock has a much lower relative contribution while the UIP shock contributes slightly more.

Turning to CPI inflation rates, the stronger impact of foreign shocks in the DSGE-VAR is counterbalanced by lower contributions from markup shocks (and to a lesser extent, efficient supply shocks) but the contribution of policy shock and demand factors is nonetheless higher. Here again, most of the structural shocks contribute to the higher spillovers, with the policy shocks having a particularly stronger impact. For open economy shocks, the UIP shock has a higher effect on inflation for both countries in the DSGE-

VAR whereas the Home bias shock contributes relatively less in the US and more in the euro area than in the DSGE.

On interest rates, notice that despite the larger role for foreign shocks, the contributions of PPI-markup and investment specific productivity shocks for both countries and the productivity shock for the US in particular, are nonetheless higher in the DSGE-VAR than in the DSGE. The international transmission on interest rates is significantly higher for most of the structural shocks in the DSGE-VAR, except for the public expenditure shocks. The home bias shock contributes less to interest rate fluctuations in the DSGE-VAR while the relative contribution of the UIP shock is higher in the US but lower in the euro area.

Concerning the nominal exchange rate, the UIP shock and the home bias shock explain around 65% of fluctuations in the DSGE-VAR against 70% in the DSGE due to a lower relative contribution of the home bias shock. This is counterbalanced by a higher effect of euro area disturbances, affecting most shocks and in particular markup shocks. While the contribution of US shock is similar in both models, this masks lower impact of US productivity and public expenditure shocks compensated by higher contributions of labor supply, preference and markup shocks. But overall, the shock decomposition of the nominal exchange rate is not strongly different between the DSGE-VAR and the DSGE. The DSGE-VAR is indeed not considerably increasing the role of domestic fundamentals in the determination of the nominal exchange rate. To precise this assessment, we estimated a version of the model where the UIP condition is replaced by an exogenous AR(1) process for the exchange rate<sup>4</sup>. In the DSGE, the nominal exchange rate volatility is mechanically determined by the exogenous AR(1) process. In the DSGE-VAR however, the other structural shocks bring a significant contribution to aggregate fluctuations but this contribution remains around 10% (not reported here). Moreover, the sign of the endogenous responses of exchange rate in the DSGE-VAR appears in some cases at odds with the implications of the UIP condition in the theoretical model. Therefore, the exchange rate remains weakly dependent on the other observed variables introduced in the estimation and, as we will see later, this could question the relative success of the UIP condition in pinning down similar initial responses exchange rate in the DSGE and in the DSGE-VAR.

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<sup>4</sup>We do not report here the estimation but it can be obtained from the authors. Note also that in the specification with exogenous exchange rate, real variables have unit root in the model and there we had to estimate with real observed variables in first difference.

We now turn to the shock decomposition of selected covariances at different time horizons. Regarding the cross-country covariance of output, the conditional covariance in the DSGE-VAR is positive at all horizons and increases continuously. In the DSGE, the covariance is almost negligible at all horizons. A first element to explain the stronger comovement of output in the DSGE-VAR relates to the international transmission of markups and policy shocks which is significantly positive beyond a two-year horizon in the DSGE-VAR while the spillovers of those shocks are negative in the DSGE. In addition, the labor supply shocks (and in particular the euro area one) contribute strongly to the aggregate comovements of output in the DSGE-VAR but they have quasi no impact on the cross-country covariance in the DSGE. Finally the open economy shocks which in the DSGE generates negative correlation between outputs, have in the DSGE-VAR a significantly higher contribution beyond a two-year horizon. However, while demand shocks are clearly associated with positive cross-country covariance in the DSGE, their effect is not reinforced in the DSGE-VAR. Most of the below-two-year impacts are lower than in the DSGE and the only shocks which have higher contributions are the US investment specific productivity and the euro area public expenditure shocks beyond a five-year horizon. Another particularly striking feature of the DSGE-VAR is the negative contribution to output comovement of the euro area preference shock beyond a two-year horizon. We will explore this further when analyzing the impulse responses. Note at last that in the DSGE already, the contribution of productivity shocks to output comovement is slightly positive for the euro area and negative for the US, notably due to the fact that only the US productivity shock is correlated to the UIP shock. The DSGE-VAR amplifies strongly this asymmetry.

Both the DSGE-VAR and the DSGE fail to capture the positive cross-country covariance of consumption and imply strongly negative unconditional comovement. However the sources of this negative covariance are quite different. In the DSGE, the main contributor is the home bias shock with the UIP shock, the preference shocks and the public expenditure shocks in the long term having a negligible negative impact. In the DSGE-VAR, the long term negative contributions are more evenly shared between home bias, UIP, preference and public expenditure shocks. Note also that the productivity shock in the euro area implies a relatively high negative long-term covariance. However, the DSGE-VAR succeeds in generating a positive conditional cross-country covariance at a two-to-three-year horizon. This results mainly from the fact that, among the negative long term contributors, the contribution of public expen-

diture shocks is positive at this horizon, and second that other shocks and in particular policy shocks have a significant positive impact.

The conditional cross-country covariance of CPI inflation rates is positive and gradually increasing in the DSGE-VAR but slightly negative in the DSGE. Actually, the DSGE induces negative comovement of those variables due mainly to the open economy and the public expenditure shocks which more than compensate the positive transmission of efficient supply shocks and markup shocks while the other disturbances have a neutral impact. To generate the positive covariance, the DSGE-VAR raises the contribution of almost all domestic shocks while leaving quasi unchanged the effect of open economy shocks. Notably, euro area labor supply, CPI markup and policy shocks contribute strongly in the DSGE-VAR. Note however, the negative transmission of euro area productivity shock compared with the positive one in the DSGE.

Finally, regarding interest rates, the DSGE-VAR implies at all horizons, higher cross-country correlation than in the DSGE. All shocks except the open economy shocks and in particular the UIP, shock induce positive comovement in the DSGE. In the DSGE-VAR however, the only negative contributions comes from the euro area productivity shock while the UIP shock is almost neutral and the home bias shock has a positive effect at all horizons. All the other shocks contribute to aggregate comovement with particularly strong effect of euro area labor supply and to a lesser extent preference and policy shocks.

All in all, the analysis of shock decomposition points to significant differences not only in the international spillovers but also in the domestic transmission of shocks between the DSGE-VAR and the DSGE. The low interdependence generated by the DSGE model may well be due to fundamental misspecifications in some transmission channels as well as the structural specifications of disturbances. The DSGE-VAR estimation also provides another piece of evidence regarding the structural sources of comovements between the US and the euro area. It seems in particular that the predominant role of demand spillovers in driving cross-country output correlation is questioned: labor supply and policy shocks for example have strong positive transmission in the DSGE-VAR. However, it is important to keep in mind that, in presence of serious model-misspecification about the international dimension, the identification scheme used in the DSGE-VAR may be induce spurious transmissions. In particular, if

most of the comovements between the US and the euro area were due to common factors, the DSGE-VAR estimation of the theoretical model which is based on structural shocks independent from one country to the other, would most likely generate high spillovers as some country-specific shocks would tend to capture the effects of common disturbances. To explore further those properties, we compare the impulse responses from the DSGE-VAR and the DSGE.

### 4.3 Misspecification and the economic propagation of shocks

As in [Del Negro et al. \(2006\)](#), we first consider the comparison of the impulse response functions between the DSGE-VAR and the DSGE as a useful model evaluation procedure. It allows us to investigate in more details the dimensions in which the model restrictions are not supported by the data. But as mentioned earlier, the DSGE-VAR model is also a structurally-identified VAR and as such, provides some empirical evidence on domestic as well as international transmissions of a wide range of macroeconomic disturbances.

Figures 4 to 21 show the median impulse response functions and the density intervals covering 80% of the posterior distribution for the DSGE-VAR and the DSGE. The impulse responses for the DSGE have obviously been computed with the same draws of DSGE mode parameter  $\theta$  that generate the DSGE-VAR impulse responses.

#### 4.3.1 Domestic transmission

First of all, we analyze the propagation mechanism of the structural shocks (except the UIP and the home bias shocks) on the source country. For the moment, we do not comment on the exchange rate adjustment and on the international spillovers. In general, the results show that, as far as the domestic transmission is concerned, the identification scheme used and the DSGE-VAR estimation provide impulse responses which are economically interpretable and qualitatively in line with the DSGE transmission. Some impulse responses are even quasi similar to the one of the DSGE-VAR. At the same time, there are some clear signs of misspecification even in the "closed-economy" transmission channels. In this respect, some of our results for the US are similar to what [Del Negro et al. \(2006\)](#) obtain. It seems

also that the euro area block suffers slightly more from misspecification than the US block.

The propagation of US productivity shock in the DSGE-VAR is very close to the one of the DSGE except for the decline of interest rate which is more pronounced in the DSGE-VAR. The same shock in the euro area propagates however very differently in both models. The DSGE-VAR response suggests a less disinflationary and less expansionary transmission to domestic variables. Turning to labor supply shocks, the US and euro area impulse responses are very close in both models except from a higher and more persistent response of hours and a stronger decline of euro area interest rate. But overall, the DSGE-VAR seems to be very much in agreement with the DSGE model concerning this shock.

In both countries, the transmission of preference shocks in the DSGE-VAR is relatively close to the DSGE for output, consumption and to a lesser extent, investment but the response of interest rate is higher, the increase of real wage is delayed and more persistent. There is a decrease in employment in the medium term in the DSGE-VAR and the inflationary effects are slightly stronger in the short term. For the US, the impulse responses to a public expenditure shock are relatively close in the DSGE-VAR and the DSGE while for the euro area, the medium term responses of interest rate, real variables and in particular employment, are lower and more persistent in the DSGE-VAR with inflation slightly higher in the short-term. For both countries, the effect of an investment specific productivity shock in the DSGE-VAR generates a more persistent decline of consumption below the baseline, a higher response of interest rate and a stronger response of inflation in the short-term.

The transmission of US PPI-markup shocks in the DSGE-VAR is more pronounced and persistent on output, consumption and employment while the interest rate increases more and longer. For the euro area, a marked difference between the DSGE-VAR and the DSGE concerns the reaction of real wages whose decline is stronger and long-lasting in the DSGE-VAR. In addition, the interest rate falls rapidly below baseline in the DSGE-VAR whereas it remains above in the DSGE. Positive CPI-markup shocks have a negative transmission to producer prices in the DSGE which is not supported by the DSGE-VAR. In both countries, the later implies some significant increase in second round effect on GDP inflation. The response of consumption is also much lower in the DSGE-VAR than in the DSGE.

Finally, we find that for both countries, the effects of monetary policy are larger and more persistent on real variables and inflation rates in the DSGE-VAR.

#### 4.3.2 International spillovers and open economy shocks

The comparison of impulse responses between the DSGE-VAR and the DSGE also complements the analysis of correlations presented before and gives a better interpretation of the differences in international spillovers.

In the DSGE, positive efficient supply shocks raise the natural output of the domestic economy, creating a slack in resource use, and call for real depreciation in order for demand to absorb the excess supply. Both monetary regimes accommodate those shocks in the source country by decreasing interest rates. Exchange rate overshoots, depreciating on impact and then gradually appreciating. Current account increases as the relative price effect overcomes the income effect. Spill-overs to the foreign output is therefore ambiguous a priori with conflicting relative price and income effects.

Regarding positive US productivity shock first, the nominal exchange rate depreciate strongly on impact, both in the DSGE-VAR and the DSGE, but then appreciate rapidly afterwards in the DGSE while the depreciation is more protracted in the DSGE-VAR. Note that for the US case, the estimated correlation of the productivity shock with the exchange rate risk premium is reinforcing the initial depreciation. The US current account increases on impact in both models but then reverts to baseline more rapidly in the DSGE-VAR. The negative spillovers on activity, hours and inflation are slightly more pronounced in the DSGE-VAR while the positive transmission to consumption, investment and interest rate is lower. But the width of the density intervals for the impulse responses suggests that such international spillovers may not be relevant in the DSGE-VAR. Concerning euro area productivity shock, the domestic transmission is already quite misspecified and the international propagation seems quite erratic and barely significant.

Another efficient supply shock in the model is the labor supply shock. The US shock is associated with a stronger depreciation of the dollar in the DSGE-VAR. On the euro area, the US labor supply shock

has different effect than in the DSGE. The spillover is indeed strongly positive on output, consumption, real wages and interest rates while the decline in inflation and employment is more pronounced. The US current account expands also much more in the DSGE-VAR. The international transmission of the euro area labor supply shock is however relatively striking in the DSGE-VAR: it generates a substantial negative response of US inflation rates and interest rate in the very short term and then induces a strong and persistent expansion of real variables. Notwithstanding the increase in US real wages increase, the DSGE-VAR impulse responses look as if the shock was common.

As we mentioned earlier, the DSGE we use in this paper does not introduce common factors or any type of cross-country correlations in structural shocks. In addition, the economic behaviors are highly symmetric between the US and the euro. Imagine now that the data are generated by the DSGE, imposing that all the euro area shocks are common with the US while all the US shocks are idiosyncratic. We estimate then a DSGE-VAR on those data but using a DSGE with a shock structure independent from one country to the other (i.e. with no common shocks). Most probably, in the identification scheme of the DSGE-VAR, the impulse responses from the euro area shocks will portray a strong transmission to the US variables while in reality, the measured spillovers are coming from common factors.

To illustrate the potential pitfalls of specifying only idiosyncratic shocks in our DSGE, we estimated the following state-space model with US and euro area detrended output as observed variables, one common i.i.d shock and two AR(1) idiosyncratic disturbances:

$$\begin{aligned}
\widehat{z}_t &= \begin{matrix} a_1 \widehat{z}_{t-1} & + & a_3 \widehat{z}_{t-1}^* & + & f_t + e_t \\ 0.98 & (0.93, 1.04) & -0.12 & (-0.18, -0.48) \end{matrix} \\
\widehat{z}_t^* &= \begin{matrix} a_4 \widehat{z}_{t-1} & + & a_2 \widehat{z}_{t-1}^* & + & f_t + e_t^* \\ 0.11 & (0.08, 0.13) & 0.92 & (0.89, 0.95) \end{matrix} \\
e_t &= \begin{matrix} \rho e_{t-1} & + & \epsilon_t \\ 0.12 & (0.00, 0.23) \end{matrix} \\
e_t^* &= \begin{matrix} \rho^* e_{t-1}^* & + & \epsilon_t^* \\ 0.50 & (0.10, 0.98) \end{matrix}
\end{aligned}$$

where  $f_t$ ,  $e_t$  and  $e_t^*$  are i.i.d. shocks. We report in subscripts the posterior mean and density intervals of the VAR(1) parameters.

The estimation shows first that the euro area idiosyncratic shock is not even identified (with the posterior distribution being identical the prior one). Therefore the euro area GDP is mainly explained by a common factor ( $f_t$ ) and the spillover from the US idiosyncratic shock ( $e_t$ ). In terms of variance decom-



position,  $\epsilon_t$  explains 80% of US GDP volatility and  $f_t$  20%. For the euro area, the spillovers contribute to 60% of GDP fluctuations and the common factor explains 40%. Regarding the dynamic responses, the US idiosyncratic shock has a spillover around 0.5 on the euro area at a 2-to-3-year horizon (see Figure 23).

Those results are obviously not to be taken at face value. But this very simple analysis of comovements between the US and the euro area indeed suggests that some common factors may be crucial to understand the US and euro area interdependence but more than this, it indicates some strong asymmetry in the role of idiosyncratic and common shocks between the US and the euro area. In the DSGE-VAR estimation of this paper, such asymmetry could be an important source of spurious spillovers.

In the case of the euro area labor supply shocks, we tried to reestimate the DSGE-VAR, replacing the euro area labor supply shock by a common labor supply shock and keeping the US idiosyncratic one. The modified impulse responses presented in Figure 22 show that the transmission of the common labor supply shock is very similar in the DSGE-VAR and the DSGE. The parameter estimates and the other impulse responses are not significantly modified by this amendment of the shock structure. This casts some doubts on the relevance of the surprisingly strong spillovers recorded in the benchmark DSGE-VAR for the euro area labor supply shock.

We turn now to the propagation of demand shocks in the benchmark DSGE-VAR. In the DSGE, positive efficient demand shocks like preference and public spending shocks increase the output gap and require a real appreciation so that lower external demand counterbalances excess domestic demand. Monetary policy leans against these shocks by increasing interest rate in the source country. Exchange rate overshoots, appreciating on impact and then gradually depreciating. Current account records a deficit given that both relative output and relative price effects worsen the external position. With such disturbances, theory would consequently clearly point to positive cross-country spillovers on output.

For the preference, investment and government spending shocks, on impact, the demand multipliers are similar between the DSGE-VAR and the DSGE: *ex post* the spillovers on economic activity between the US and the euro area are close to 0.1 with the preference and investment specific technology shocks and 0.2 to 0.5 with the government expenditure shocks (the later notably having a higher cross-country impact due to the correlations we introduced with the UIP shock). Therefore, in the very short term, the DSGE-VAR is already not pointing to stronger transmission on output.

In the case of government spending shocks, beyond the first quarters, such positive spillovers on output

fade away much more rapidly in the DSGE-VAR. But overall, the impulse responses, in particular for exchange rate, current account and foreign interest rate, remain relatively close between the DSGE-VAR and the DSGE, compared with the other demand shocks.

For the investment and preference shocks, there are indeed clear signs of misspecification in the international transmission, and more specially for shocks emanating from the euro area. The spillovers on output are surprisingly negative in the medium run except for the US preference shock. Another striking feature is the much stronger positive response of foreign inflation rates and interest rates in the DSGE-VAR, which is particularly pronounced for euro area shocks. This effect seems to generate most of the negative effect of real variables in the DSGE-VAR transmission and could be partly related to the sharper adjustment of the nominal exchange rate. In order to investigate further the role of the exchange rate, we estimated two other versions of the model, one without the nominal exchange rate as an observed variable and one with a DSGE specification where the UIP condition is replaced by an exogenous AR(1) process for the exchange rate (as we mentioned earlier). It turns out that both models feature admittedly some smaller discrepancies between the DSGE-VAR and the DSGE response of inflation rates in the foreign country but the stronger response of foreign interest rate as well as the high negative spillovers on real variables in the medium term are left unchanged. This clearly suggests that the transmission of the demand shocks or the cross-country correlations of such disturbances are not appropriately specified in our model. Contrary to what we investigated for the labor supply shocks for which amending the commonality of the shock structure seemed quite promising, we do not see any easy model amendment at this stage to improve this dimension and we leave that for further research. Considering inefficient shocks, in the DSGE, PPI-markup shocks on the products induce a gradual depreciation of the nominal exchange rate but the current account decreases slightly as the impact of the cost-push shock on competitiveness dominates in the short run. In terms of spill-overs, the transmission of the price-markup shock is negative on real variables and positive on inflation. By contrast, in the DSGE-VAR, the short-term international transmission of PPI-markup shocks is positive on output, with the contraction of activity being particularly strong in the euro area. The response of inflation abroad is stronger than in the DSGE. Regarding exchange rate, the DSGE-VAR implies a substantial appreciation on impact contrary to the DSGE. The interest response abroad in the DSGE-VAR is higher and more persistent in the case of a US shock and lower and more persistent in the case of a euro area shock. In the medium term, foreign real quantities expand after a euro area shock but fall below baseline after a

US shock. But overall, also taking into account the width of the density intervals, there is clearer evidence of misspecified and stronger spillovers in the case of euro area shock. However, some aspects of commonality, such as oil price shock for example, may potentially be captured in the identification scheme for this shock.

In the DGSE, the open economy dimension adds the exchange rate channel to the monetary policy transmission mechanism. The interest rate increases trigger an instantaneous appreciation of the exchange rate followed by a gradual depreciation. The net impact on the current account is a priori ambiguous as income and relative price effects play on opposite directions. So is the transmission to the foreign economy which depends crucially on the magnitude of the expenditure-switching effect and the income-absorption effect. The relative role of the different channels is obviously very sensitive to the assumptions on international price setting, the intratemporal elasticity of substitution or the degree of home bias. On net, in the DSGE, the current account deteriorates on impact and there is a small positive international spillover to foreign output, inflation and interest rate while foreign domestic demand marginal declines. In the DSGE-VAR, the transmission is also slightly positive in the very short-term but rapidly, foreign output contracts and decreases significantly below-baseline. Similarly the foreign domestic demand follows a much stronger and protracted decline in the DSGE-VAR. Those development are particularly pronounced in the case of the US monetary policy shock. In this case, the substantial depressive impact on euro area activity may be partly due to higher inflationary impact, higher interest rate increase as well as stronger appreciation of the dollar than in the DSGE. Conversely from the US monetary policy shock, a euro area monetary innovation in the DSGE-VAR implies an exchange rate reaction more consistent with the DSGE and does not generate significant short-term inflationary pressures in the US but it similarly induces an increase in US interest rate. This could partially explain why the negative transmission to foreign real variables is much more muted than in the case of a US monetary policy shock.

In the DSGE model, the UIP shock leads to a strong appreciation of the nominal exchange rate on impact and a sharp deterioration of the current account. Under both policy regimes, the appreciation is accompanied by a decrease in home interest rate and an increase of a similar magnitude in the foreign country. Over the first quarters, home output contracts and foreign output expands while home domestic demand increases and foreign domestic demand drops by a similar amount. Inflation and interest rate decrease at home but increase in the foreign country. This strong asymmetry is present with both

the DSGE and the DSGE-VAR. However, some differences emerge. Even if the short-term impact on real variables is relatively similar, the DSGE-VAR implies a stronger and more persistent response of consumption in both countries and of investment in the US, while the short-term effects on output are reversed rapidly in the DGSE-VAR, generating in the medium-term an apparent comovement between production and domestic demand. Regarding inflation, the dollar appreciation seems to have higher short-term effects, in particular for producer-price and for the euro area. The US interest rate remains longer below baseline in the DSGE-VAR whereas the EA interest rate declines rapidly below baseline after the initial increase. Finally, the deterioration of US current account following the appreciation is slightly shifted downwards in the DSGE-VAR.

The home bias shock is equivalent to a fully asymmetric world demand shock, when we abstract from the correlation with the euro area government spending shock that we introduced for the estimation. In that case, as for the UIP shock, in one country the current account increases, output expands, interest rate rises and domestic demand contracts while macro variables in the other country mirror these developments on the negative side. However, compared with the UIP shock, the exchange rate appreciates for the country experiencing the net-trade expansion. The correlation of the euro area government expenditure shock is meant to break this asymmetry and control for non-euro area external disturbances which affect the US current account. Those properties are relatively well portrayed in the DSGE-VAR. Specifically however, note that the response of euro area appears more pronounced in the DSGE-VAR suggesting again that the exchange rate pass-through, in particular to PPI inflation, could be higher than in the model.

## 5 Conclusion

In this paper, we applied the methodology developed by [Del Negro and Schorfheide \(2004\)](#) to assess the potential misspecifications in the international spillovers between the US and the euro area as described by a two-country DSGE similar to [Adjemian et al. \(2007\)](#). In the theoretical model, we abstract from common sources of fluctuations so that interdependence between the US and the euro area can only come from spillovers of idiosyncratic shocks. The main contributions of the paper cover first a methodological dimension. we improved on [Del Negro et al. \(2006\)](#) by jointly estimating the prosterior distribution of the parameter driving the weight to put on the DSGE in the DSGE-VAR estimation. In

addition, our two country framework extends the results of [Del Negro et al. \(2006\)](#) to the euro area: some of our results for the US are indeed similar to theirs and we present some evidence that the euro area block may suffer slightly more from misspecification than the US block.

Regarding the international spillovers, we show first that the international transmission on activity from demand shocks which are supposed to be the strongest given our theoretical description of the economies, are not strengthened by the DSGE-VAR estimation in the short to medium term. At the same time, monetary policy contractions seem to have a pronounced negative impact on foreign activity and more specially in the case of the US monetary policy shock. We also find a surprisingly strong positive spillover of euro area labor supply shock. But our analysis suggests that this spillover may not be economically relevant and that, in the identification scheme, the euro area labor supply shock may be capturing the transmission of a common supply factors. Finally, the transmission of the UIP shock in the DSGE-VAR points to a much and persistent impact on domestic demand as well as a higher pass-through on CPI and PPI inflation rates.

Going forward, more research is obviously needed to provide better microfoundations of US euro area interdependence coming from international spillovers. At the same time, the DSGE-VAR methodology could also be generalized to any state-space from which would allow to introduce more shocks than observed variables and therefore properly test for common factors in the structural shocks.

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## A Model description

We present thereafter the first order conditions associated with the decision problems of section 2 and close the model.

### A.1 Households

The first order condition related to consumption expenditures is given by

$$\Lambda_t = \varepsilon_t^B (C_t - hC_{t-1})^{-\sigma_C} - \beta h \mathbb{E}_t \left[ \varepsilon_{t+1}^B (C_{t+1} - hC_t)^{-\sigma_C} \right] \quad (3)$$

where  $\frac{\Lambda_t}{(1+\tau_{C,t})}$  is the lagrange multiplier associated with the budget constraint.

First order conditions corresponding to the quantity of contingent bonds imply that

$$\Lambda_t = R_t \beta \mathbb{E}_t \left[ \Lambda_{t+1} \frac{P_t}{P_{t+1}} \right] \quad (4)$$

$$\Lambda_t = R_t^* \varepsilon_t^{\Delta S} \Psi \left( \frac{\mathbb{E}_t S_{t+1}}{S_{t-1}} - 1, \frac{S_t (B_{F,t} - \bar{B}_F)}{\underline{P}_t} \right) \beta \mathbb{E}_t \left[ \Lambda_{t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}} \right]$$

where  $R_t$  and  $R_t^*$  are one-period-ahead nominal interest rates for country  $H$  and  $F$  respectively.

The previous equations imply an arbitrage condition on bond prices which corresponds to a modified uncovered interest rate parity (UIP):

$$\frac{R_t}{R_t^* \varepsilon_t^{\Delta S} \Psi \left( \frac{\mathbb{E}_t S_{t+1}}{S_{t-1}} - 1, \frac{S_t (B_{F,t} - \bar{B}_F)}{\underline{P}_t} \right)} = \frac{\mathbb{E}_t \left[ \Lambda_{t+1} \frac{S_{t+1} P_t}{S_t P_{t+1}} \right]}{\mathbb{E}_t \left[ \Lambda_{t+1} \frac{P_t}{P_{t+1}} \right]} \quad (5)$$

where  $\varepsilon_t^{\Delta S}$  is a unitary-mean disturbance affecting the risk premium.

The choices for investment, capacity utilization and capital stock result in the following first order conditions:

$$Q_t = \mathbb{E}_t \left[ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{1 + \tau_{C,t}}{1 + \tau_{C,t+1}} (Q_{t+1}(1 - \delta) + R_{t+1}^k u_{t+1} - \Phi(u_{t+1})) \right] \quad (6)$$

$$Q_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - \frac{I_t}{I_{t-1}} S' \left( \frac{I_t}{I_{t-1}} \right) \right] \varepsilon_t^I \quad (7)$$

$$+ \beta \mathbb{E}_t \left[ Q_{t+1} \frac{\Lambda_{t+1}}{\Lambda_t} \frac{1 + \tau_{C,t}}{1 + \tau_{C,t+1}} \left( \frac{I_{t+1}}{I_t} \right)^2 S' \left( \frac{I_{t+1}}{I_t} \right) \varepsilon_{t+1}^I \right] = 1$$

$$R_t^k = \Phi'(u_t) \quad (8)$$

where  $\frac{\Lambda_t}{(1+\tau_{C,t})} Q_t$  is the lagrange multiplier associated with the capital accumulation equation

$$K_t = (1 - \delta) K_{t-1} + \varepsilon_t^I \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t \quad (9)$$

The first order condition of the wage-setting program can be written recursively as follows:

$$\frac{\widetilde{W}_t(h)}{P_t} = \left( \mu_w \frac{\mathcal{H}_{1,t}^w}{\mathcal{H}_{2,t}^w} \right)^{\frac{\mu_w - 1}{\mu_w(1+\sigma_L) - 1}}$$

$$\mathcal{H}_{1,t}^w = \varepsilon_t^B \varepsilon_t^L \widetilde{L} L_t^{1+\sigma_L} \left[ \frac{w_t}{1+\tau_{C,t}} \right]^{\frac{(1+\sigma_L)\mu_w}{\mu_w - 1}} + \alpha_w \beta \mathbb{E}_t \left[ \left( \frac{\Pi_{t+1}}{\Pi_t^{\xi_w} \overline{\Pi}^{1-\xi_w}} \right)^{\frac{(1+\sigma_L)\mu_w}{\mu_w - 1}} \mathcal{H}_{1,t+1}^w \right] \quad (10)$$

$$\mathcal{H}_{2,t}^w = (1 - \tau_w) \Lambda_t L_t \left[ \frac{w_t}{1+\tau_{C,t}} \right]^{\frac{\mu_w}{\mu_w - 1}} + \alpha_w \beta \mathbb{E}_t \left[ \left( \frac{\Pi_{t+1}}{\Pi_t^{\xi_w} \overline{\Pi}^{1-\xi_w}} \right)^{\frac{1}{\mu_w - 1}} \mathcal{H}_{2,t+1}^w \right] \quad (11)$$

where  $w_t$  denotes the aggregate real wage (measured with the before-tax CPI).

Finally, the aggregate wage dynamics is given by.

$$\left[ \frac{w_t}{1+\tau_{C,t}} \right]^{\frac{1}{1-\mu_w}} = (1 - \alpha_w) \left( \mu_w \frac{\mathcal{H}_{1,t}^w}{\mathcal{H}_{2,t}^w} \right)^{-\frac{1}{\mu_w(1+\sigma_L) - 1}} + \alpha_w \left[ \frac{w_{t-1}}{1+\tau_{C,t-1}} \right]^{\frac{1}{1-\mu_w}} \left( \frac{\Pi_t}{\Pi_{t-1}^{\xi_w} \overline{\Pi}^{1-\xi_w}} \right)^{\frac{-1}{1-\mu_w}} \quad (12)$$

## A.2 Firms

The real marginal cost is identical across intermediate producers within each country:

$$MC_t = \frac{w_t^{(1-\alpha)} R_t^{k\alpha}}{\varepsilon_t^A \alpha^\alpha (1-\alpha)^{(1-\alpha)} T_{H,t}} \quad (13)$$

$$MC_t^* = \frac{W_t^{*(1-\alpha)} R_t^{k*\alpha}}{\varepsilon_t^{A*} \alpha^\alpha (1-\alpha)^{(1-\alpha)} T_F^*} \quad (14)$$

The first order condition associated with the firm's choice of  $\hat{p}_t(h)$  for local sales is  $\frac{\hat{p}_t(h)}{P_{H,t}} = \mu \frac{\mathcal{Z}_{H1,t}}{\mathcal{Z}_{H2,t}}$  where

$$\mathcal{Z}_{H1,t} = \Lambda_t MC_t Y_{H,t} \frac{T_{H,t}}{1+\tau_{C,t}} + \alpha_H \beta \mathbb{E}_t \left[ \left( \frac{\Pi_{H,t+1}}{\Pi_{H,t}^{\gamma_H} \overline{\Pi}^{1-\gamma_H}} \right)^{\frac{\mu}{\mu-1}} \mathcal{Z}_{H1,t+1} \right] \quad (15)$$

and

$$\mathcal{Z}_{H2,t} = (1 - \tau_t) \Lambda_t Y_{H,t} \frac{T_{H,t}}{1+\tau_{C,t}} + \alpha_H \beta \mathbb{E}_t \left[ \left( \frac{\Pi_{H,t+1}}{\Pi_{H,t}^{\gamma_H} \overline{\Pi}^{1-\gamma_H}} \right)^{\frac{1}{\mu-1}} \mathcal{Z}_{H2,t+1} \right] \quad (16)$$

Accordingly, the aggregate price dynamics leads to the following relation.

$$1 = \alpha_H \left( \frac{\Pi_{H,t}}{\Pi_{H,t-1}^{\gamma_H} \overline{\Pi}^{1-\gamma_H}} \right)^{\frac{1}{\mu-1}} + (1 - \alpha_H) \left( \mu \frac{\mathcal{Z}_{H1,t}}{\mathcal{Z}_{H2,t}} \right)^{\frac{1}{1-\mu}} \quad (17)$$

Equations analogous hold for foreign producers and governs the dynamics of  $\Pi_{F,t}^*$  as follows

$$\mathcal{Z}_{F1,t}^* = \Lambda_t^* MC_t^* Y_{F,t}^* \frac{T_{F,t}^*}{1+\tau_{C,t}^*} + \alpha_F^* \beta \mathbb{E}_t \left[ \left( \frac{\Pi_{F,t+1}^*}{\Pi_{F,t}^{*\gamma_F} \overline{\Pi}^{1-\gamma_F^*}} \right)^{\frac{\mu}{\mu-1}} \mathcal{Z}_{F1,t+1}^* \right] \quad (18)$$



$$\mathcal{Z}_{F2,t}^* = (1 - \tau_t^*) \Lambda_t^* Y_{F,t}^* \frac{T_{F,t}^*}{1 + \tau_{C,t}^*} + \alpha_F^* \beta \mathbb{E}_t \left[ \left( \frac{\Pi_{F,t+1}^*}{\tilde{\Pi}_{F,t}^{*\gamma_F^*} \bar{\Pi}^{*1-\gamma_F^*}} \right)^{\frac{1}{\mu-1}} \mathcal{Z}_{F2,t+1}^* \right] \quad (19)$$

and

$$1 = \alpha_F^* \left( \frac{\Pi_{F,t}^*}{\tilde{\Pi}_{F,t-1}^{*\gamma_F^*} \bar{\Pi}^{*1-\gamma_F^*}} \right)^{\frac{1}{\mu-1}} + (1 - \alpha_F^*) \left( \mu \frac{\mathcal{Z}_{F1,t}^*}{\mathcal{Z}_{F2,t}^*} \right)^{\frac{1}{1-\mu}} \quad (20)$$

The inflation dynamics of LCP export prices for the country  $H$ ,  $\tilde{\Pi}_{H,t}^*$ , is described by the following three equations

$$\tilde{\mathcal{Z}}_{H1,t}^* = \Lambda_t MC_t Y_{H,t}^* \frac{T_{H,t}}{1 + \tau_{C,t}} + \alpha_F^* \beta \mathbb{E}_t \left[ \left( \frac{\tilde{\Pi}_{H,t+1}^*}{\tilde{\Pi}_{H,t}^{*\gamma_F^*} \bar{\Pi}^{*1-\gamma_F^*}} \right)^{\frac{\mu}{\mu-1}} \tilde{\mathcal{Z}}_{H1,t+1}^* \right] \quad (21)$$

$$\tilde{\mathcal{Z}}_{H2,t}^* = (1 - \tau_t) \Lambda_t Y_{H,t}^* \frac{T_{H,t}}{1 + \tau_{C,t}} R \tilde{E} R_{H,t} + \alpha_F^* \beta \mathbb{E}_t \left[ \left( \frac{\tilde{\Pi}_{H,t+1}^*}{\tilde{\Pi}_{H,t}^{*\gamma_F^*} \bar{\Pi}^{*1-\gamma_F^*}} \right)^{\frac{1}{\mu-1}} \tilde{\mathcal{Z}}_{H2,t+1}^* \right] \quad (22)$$

$$1 = \alpha_F^* \left( \frac{\tilde{\Pi}_{H,t}^*}{\tilde{\Pi}_{H,t-1}^{*\gamma_F^*} \bar{\Pi}^{*1-\gamma_F^*}} \right)^{\frac{1}{\mu-1}} + (1 - \alpha_F^*) \left( \mu \frac{\tilde{\mathcal{Z}}_{H1,t}^*}{\tilde{\mathcal{Z}}_{H2,t}^*} \right)^{\frac{1}{1-\mu}} \quad (23)$$

LCP export price inflation for country  $F$ ,  $\tilde{\Pi}_{F,t}$ , is given by the equivalent formulation

$$\tilde{\mathcal{Z}}_{F1,t} = \Lambda_t^* MC_t^* Y_{F,t}^* \frac{T_{F,t}^*}{1 + \tau_{C,t}^*} + \alpha_H \beta \mathbb{E}_t \left[ \left( \frac{\tilde{\Pi}_{F,t+1}}{\tilde{\Pi}_{F,t}^{\gamma_H} \bar{\Pi}^{*1-\gamma_H}} \right)^{\frac{\mu}{\mu-1}} \tilde{\mathcal{Z}}_{F1,t+1} \right] \quad (24)$$

$$\tilde{\mathcal{Z}}_{F2,t} = (1 - \tau_t^*) \Lambda_t^* Y_{F,t}^* \frac{T_{F,t}^*}{1 + \tau_{C,t}^*} R \tilde{E} R_{F,t} + \alpha_H \beta \mathbb{E}_t \left[ \left( \frac{\tilde{\Pi}_{F,t+1}}{\tilde{\Pi}_{F,t}^{\gamma_H} \bar{\Pi}^{*1-\gamma_H}} \right)^{\frac{1}{\mu-1}} \tilde{\mathcal{Z}}_{F2,t+1} \right] \quad (25)$$

$$1 = \alpha_H \left( \frac{\tilde{\Pi}_{F,t}}{\tilde{\Pi}_{F,t-1}^{\gamma_H} \bar{\Pi}^{*1-\gamma_H}} \right)^{\frac{1}{\mu-1}} + (1 - \alpha_H) \left( \mu \frac{\tilde{\mathcal{Z}}_{F1,t}}{\tilde{\mathcal{Z}}_{F2,t}} \right)^{\frac{1}{1-\mu}} \quad (26)$$

Moreover, cost minimization implies that capital labor ratio are equalized across firms in each country. Aggregate capital labor ratios are therefore given by

$$\frac{w_t L_t}{R_t^k u_t K_{t-1}} = \frac{1 - \alpha}{\alpha} \quad (27)$$

and

$$\frac{w_t^* L_t^*}{R_t^{k*} u_t^* K_{t-1}^*} = \frac{1 - \alpha}{\alpha} \quad (28)$$

### A.3 Market clearing conditions

Aggregate domestic demands are given by

$$Y_t = C_t + I_t + \overline{G}\varepsilon_t^G + \Phi(u_t)K_{t-1} \quad (29)$$

$$Y_t^* = C_t^* + I_t^* + \overline{G}\varepsilon_t^{G*} + \Phi(u_t^*)K_{t-1}^* \quad (30)$$

where  $K_t$  and  $K_t^*$  are the aggregate capital stocks.

Aggregate productions verify

$$Z_t = \varepsilon_t^A (u_t K_{t-1})^\alpha (L_t)^{1-\alpha} - \Omega \quad (31)$$

$$Z_t^* = \varepsilon_t^{A*} (u_t^* K_{t-1}^*)^\alpha (L_t^*)^{1-\alpha} - \Omega \quad (32)$$

where  $L_t$  and  $L_t^*$  are the labour input.

Market clearing conditions in goods markets lead to the following relations

$$Z_t = n_t \Delta_{H,t} (T_{H,t})^{-\xi} Y_t + (1 - n_t^*) \Delta_{H,t}^* \left( \frac{T_{F,t}^*}{T_t^*} \right)^{-\xi} Y_t^* \quad (33)$$

$$Z_t^* = n_t^* \Delta_{F,t}^* (T_{F,t}^*)^{-\xi} Y_t^* + (1 - n_t) \Delta_{F,t} (T_t T_{H,t})^{-\xi} Y_t \quad (34)$$

where  $\Delta_{H,t} = \int_0^1 \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\frac{\mu}{\mu-1}} dh$ ,  $\Delta_{H,t}^* = \int_0^1 \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\frac{\mu}{\mu-1}} dh$ ,  $\Delta_{F,t} = \int_0^1 \left( \frac{p_t^*(f)}{P_{F,t}} \right)^{-\frac{\mu}{\mu-1}} df$  and  $\Delta_{F,t}^* = \int_0^1 \left( \frac{p_t(f)}{P_{F,t}^*} \right)^{-\frac{\mu}{\mu-1}} df$  measure price dispersions among products of country  $H$  and  $F$ , sold locally or exported. Those indexes have the following dynamics

$$\Delta_{H,t} = (1 - \alpha_H) \left( \mu \frac{Z_{H1,t}}{Z_{H2,t}} \right)^{-\frac{\mu}{\mu-1}} + \alpha_H \Delta_{H,t-1} \left( \frac{\Pi_{H,t}}{\Pi_{H,t-1}^{\gamma_H} \overline{\Pi}^{1-\gamma_H}} \right)^{\frac{\mu}{\mu-1}} \quad (35)$$

$$\Delta_{F,t}^* = (1 - \alpha_F^*) \left( \mu \frac{Z_{F1,t}^*}{Z_{F2,t}^*} \right)^{-\frac{\mu}{\mu-1}} + \alpha_F^* \Delta_{F,t-1}^* \left( \frac{\Pi_{F,t}^*}{\Pi_{F,t-1}^{*\gamma_F} \overline{\Pi}^{*1-\gamma_F}} \right)^{\frac{\mu}{\mu-1}} \quad (36)$$

$$\Delta_{H,t}^* = \eta \Delta_{H,t} + (1 - \eta) \tilde{\Delta}_{H,t}^* \quad (37)$$

$$\tilde{\Delta}_{H,t}^* = (1 - \alpha_F^*) \left( \mu \frac{\tilde{Z}_{H1,t}}{\tilde{Z}_{H2,t}} \right)^{-\frac{\mu}{\mu-1}} + \alpha_F^* \tilde{\Delta}_{H,t-1}^* \left( \frac{\tilde{\Pi}_{H,t}^*}{\tilde{\Pi}_{H,t-1}^{*\gamma_F} \overline{\Pi}^{*1-\gamma_F}} \right)^{\frac{\mu}{\mu-1}} \quad (38)$$

$$\Delta_{F,t} = \eta^* \Delta_{F,t}^* + (1 - \eta^*) \Delta_{F,t} \quad (39)$$

$$\tilde{\Delta}_{F,t} = (1 - \alpha_H) \left( \mu \frac{\tilde{Z}_{H1,t}}{\tilde{Z}_{H2,t}} \right)^{-\frac{\mu}{\mu-1}} + \alpha_H \tilde{\Delta}_{F,t} \left( \frac{\tilde{\Pi}_{F,t}}{\tilde{\Pi}_{F,t-1}^{\gamma_H} \overline{\Pi}^{1-\gamma_H}} \right)^{\frac{\mu}{\mu-1}} \quad (40)$$

Equilibrium in the bond markets implies that  $B_{F,t} + B_{F,t}^* = 0$  and  $B_{H,t} + B_{H,t}^* = 0$ . Moreover, demand for bonds denominated in currency  $F$  emanating from agents in country  $H$  is given by

$$\begin{aligned} \frac{S_t B_{F,t}}{P_t R_t^*} - \frac{B_{H,t}^*}{P_t R_t} &= \frac{S_t B_{F,t-1}}{P_t} - \frac{B_{H,t-1}^*}{P_t} \\ &+ T_{H,t} Y_{H,t} + \underline{RER}_t \frac{T_{F,t}^*}{T_t^*} Y_{H,t}^* - Y_t \end{aligned} \quad (41)$$

where  $\underline{RER}_t$  is the real exchange rate measured with distribution prices gross of consumption taxes.

Let us define the current account of country  $H$  as  $CA_t = \frac{S_t(B_{F,t} - B_{F,t-1})}{P_t R_t^*} - \frac{(B_{H,t}^* - B_{H,t-1}^*)}{P_t R_t}$ .

Some relative prices have finally to be defined as a function of stationary variables. First, the 4 inflation rates for export prices and local sales prices determine 3 relative prices: 2 relative export margins for LCP producers and interior terms of trade for country  $H$ .

$$R\tilde{E}R_{H,t} = R\tilde{E}R_{H,t-1} \frac{\tilde{\Pi}_{H,t}^* (1 + \Delta S_t)}{\Pi_{H,t}} \quad (42)$$

$$R\tilde{E}R_{F,t} = R\tilde{E}R_{H,t-1} \frac{\tilde{\Pi}_{F,t}}{\Pi_{F,t}^* (1 + \Delta S_t)} \quad (43)$$

$$T_t = T_{t-1} \frac{\Pi_{F,t}}{\Pi_{H,t}} \quad (44)$$

The following variables are deduced from the previous three relative prices.

$$RER_{H,t} = \left[ \eta + (1 - \eta) R\tilde{E}R_{H,t}^{\frac{1}{1-\mu}} \right]^{1-\mu} \quad (45)$$

$$RER_{F,t} = \left[ \eta + (1 - \eta) R\tilde{E}R_{F,t}^{\frac{1}{1-\mu}} \right]^{1-\mu} \quad (46)$$

$$T_t^* = \frac{T_t}{RER_{H,t} RER_{F,t}} \quad (47)$$

$$T_{H,t} = \left[ n_t + (1 - n_t) T_t^{1-\xi} \right]^{\frac{1}{\xi-1}} \quad (48)$$

$$T_{F,t}^* = \left[ n_t^* + (1 - n_t^*) T_t^{*\xi-1} \right]^{\frac{1}{\xi-1}} \quad (49)$$

$$\underline{RER}_t = RER_{H,t} T_{H,t} \frac{T_t^*}{T_{F,t}^*} \quad (50)$$

Finally, aggregate export price inflation rates and after-tax CPI inflation rates are given by

$$\Pi_{H,t}^* = \frac{RER_{H,t}}{RER_{H,t-1}} \frac{\Pi_{H,t}}{(1 + \Delta S_t)} \quad (51)$$

$$\Pi_{F,t} = \frac{RER_{F,t}}{RER_{F,t-1}} \Pi_{F,t}^* (1 + \Delta S_t) \quad (52)$$

$$\Pi_t = \frac{T_{H,t}}{T_{H,t-1}} \Pi_{H,t} \varepsilon_t^{CPI} \quad (53)$$

$$\Pi_t^* = \frac{T_{F,t}^*}{T_{F,t-1}^*} \Pi_{F,t}^* \varepsilon_t^{CPI*} \quad (54)$$

The model is closed by specifying the behaviour of monetary authorities:

$$r_t = \rho r_{t-1} + (1 - \rho) [r_\pi \pi_{t-1} + r_y z_{t-1}] + r_{\Delta\pi} \Delta\pi_t + r_{\Delta y} \Delta z_t + \log(\varepsilon_t^{R*}) \quad (55)$$

$$r_t^* = \rho^* r_{t-1}^* + (1 - \rho^*) [r_\pi^* \pi_{t-1}^* + r_y^* z_{t-1}^*] + r_{\Delta\pi}^* \Delta\pi_t^* + r_{\Delta y}^* \Delta z_t^* + \log(\varepsilon_t^{R*}) \quad (56)$$

## A.4 Competitive equilibrium

The competitive equilibrium is a set of stationary 27 processes for country  $H$ ,  $u_t$ ,  $Q_t$ ,  $I_t$ ,  $K_t$ ,  $R_t^k$ ,  $Y_t$ ,  $Z_t$ ,  $C_t$ ,  $\Lambda_t$ ,  $L_t$ ,  $MC_t$ ,  $\Pi_t$ ,  $\Pi_{H,t}$ ,  $\Delta_{H,t}$ ,  $\mathcal{Z}_{H1,t}$ ,  $\mathcal{Z}_{H2,t}$ ,  $\Pi_{H,t}^*$ ,  $\tilde{\Pi}_{H,t}^*$ ,  $\tilde{\Delta}_{H,t}^*$ ,  $\tilde{\mathcal{Z}}_{H1,t}^*$ ,  $\tilde{\mathcal{Z}}_{H2,t}^*$ ,  $w_t$ ,  $\mathcal{H}_{1,t}^w$ ,  $\mathcal{H}_{2,t}^w$ ,  $\Delta_{H,t}^*$ ,  $B_{F,t}$ ,  $R_t$ ,

as well as the analogous 27 processes for country  $F$ , 9 relative prices  $R\tilde{E}R_{H,t}$ ,  $R\tilde{E}R_{F,t}$ ,  $RER_{H,t}$ ,  $RER_{F,t}$ ,  $\underline{RER}_t$ ,  $T_t$ ,  $T_t^*$ ,  $T_{H,t}$ ,  $T_{F,t}^*$  and the depreciation rate  $\Delta S_t$ . The 64 stationary processes satisfy the relations (1)-(10) and their analogous for country  $F$  and the relations (11)-(54), given traditional closed-economy exogenous stochastic processes for country  $H$ ,  $\varepsilon_t^A$ ,  $\varepsilon_t^B$ ,  $\varepsilon_t^I$ ,  $\varepsilon_t^G$ ,  $\varepsilon_t^L$ ,  $\varepsilon_t^P$ ,  $\varepsilon_t^R$ , with the analogous shocks for country  $F$ , the additional open-economy exogenous stochastic processes  $\varepsilon_t^{CPI^*}$ ,  $\varepsilon_t^{CPI}$ ,  $\varepsilon_t^{\Delta S}$ ,  $\varepsilon_t^{\Delta n}$ , and initial conditions for country  $H$ ,  $C_{-1}$ ,  $I_{-1}$ ,  $K_{-1}$ ,  $\Delta_{H,-1}$ ,  $\tilde{\Delta}_{H,-1}^*$ ,  $\Pi_{H,-1}$ ,  $\tilde{\Pi}_{H,-1}^*$ ,  $w_{-1}$ , analogous initial conditions for country  $F$ , and  $R\tilde{E}R_{H,-1}$ ,  $R\tilde{E}R_{F,-1}$ ,  $T_{-1}$ .

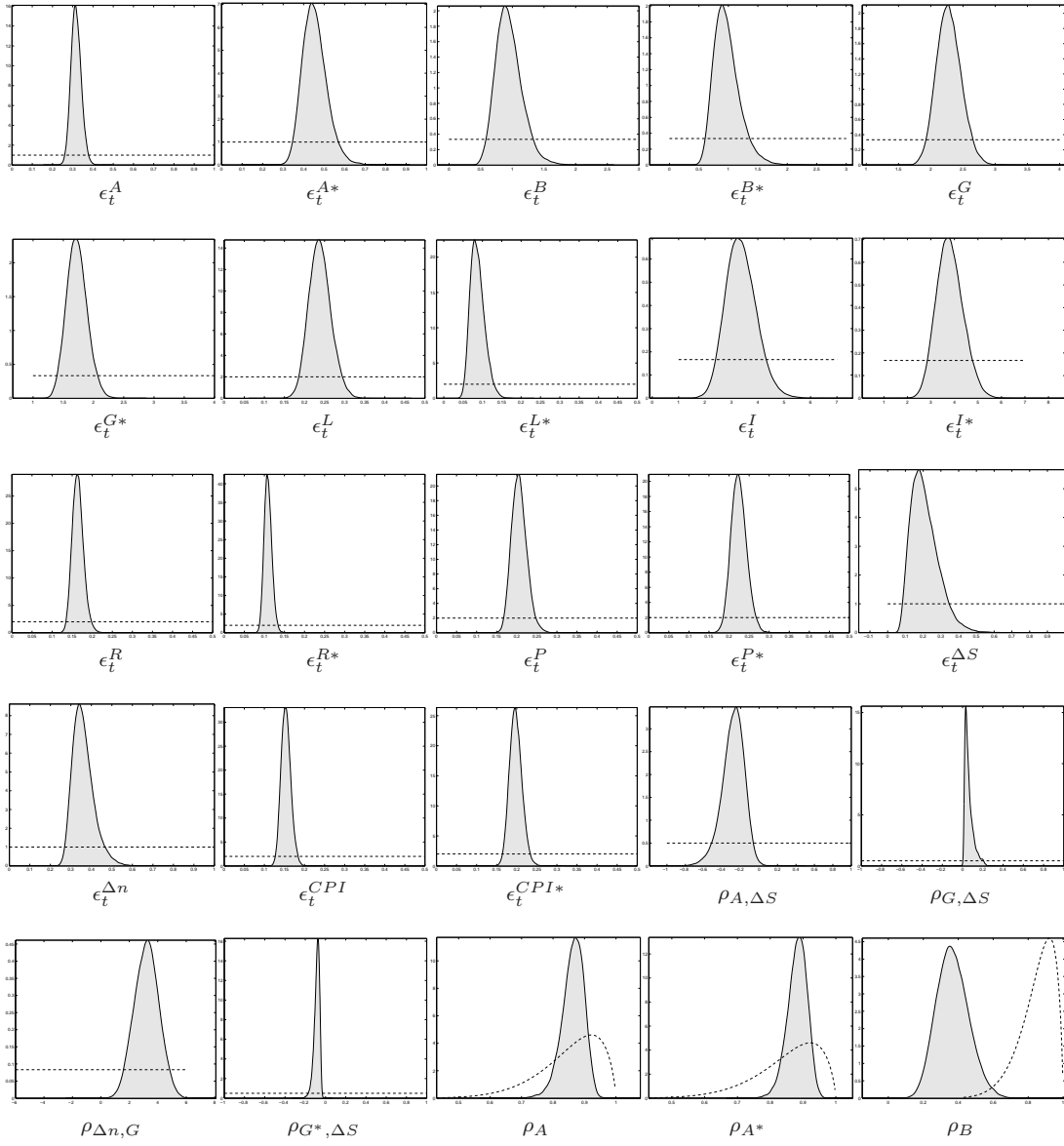


Fig. 1: Posterior densities.

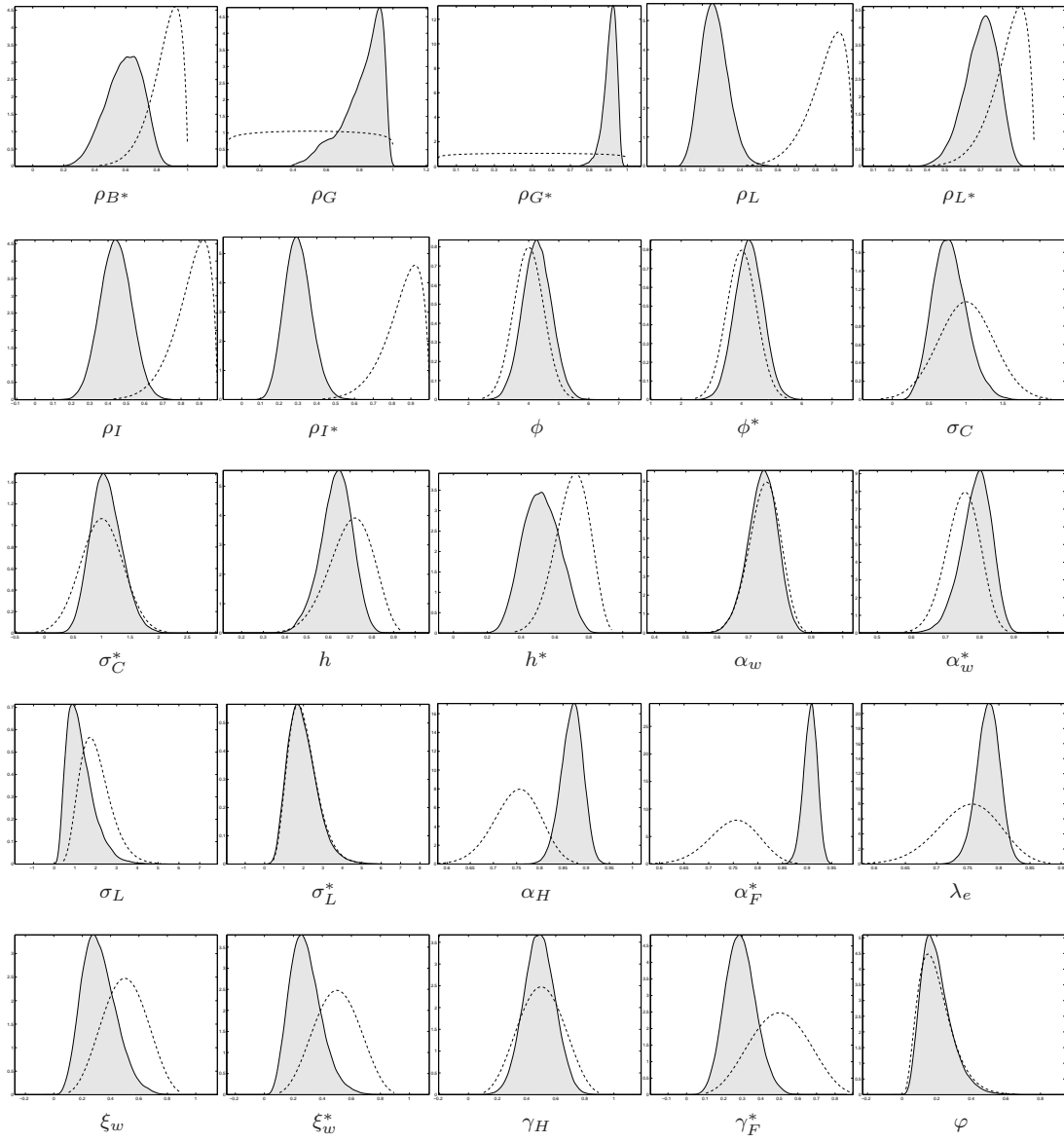


Fig. 2: Posterior densities.

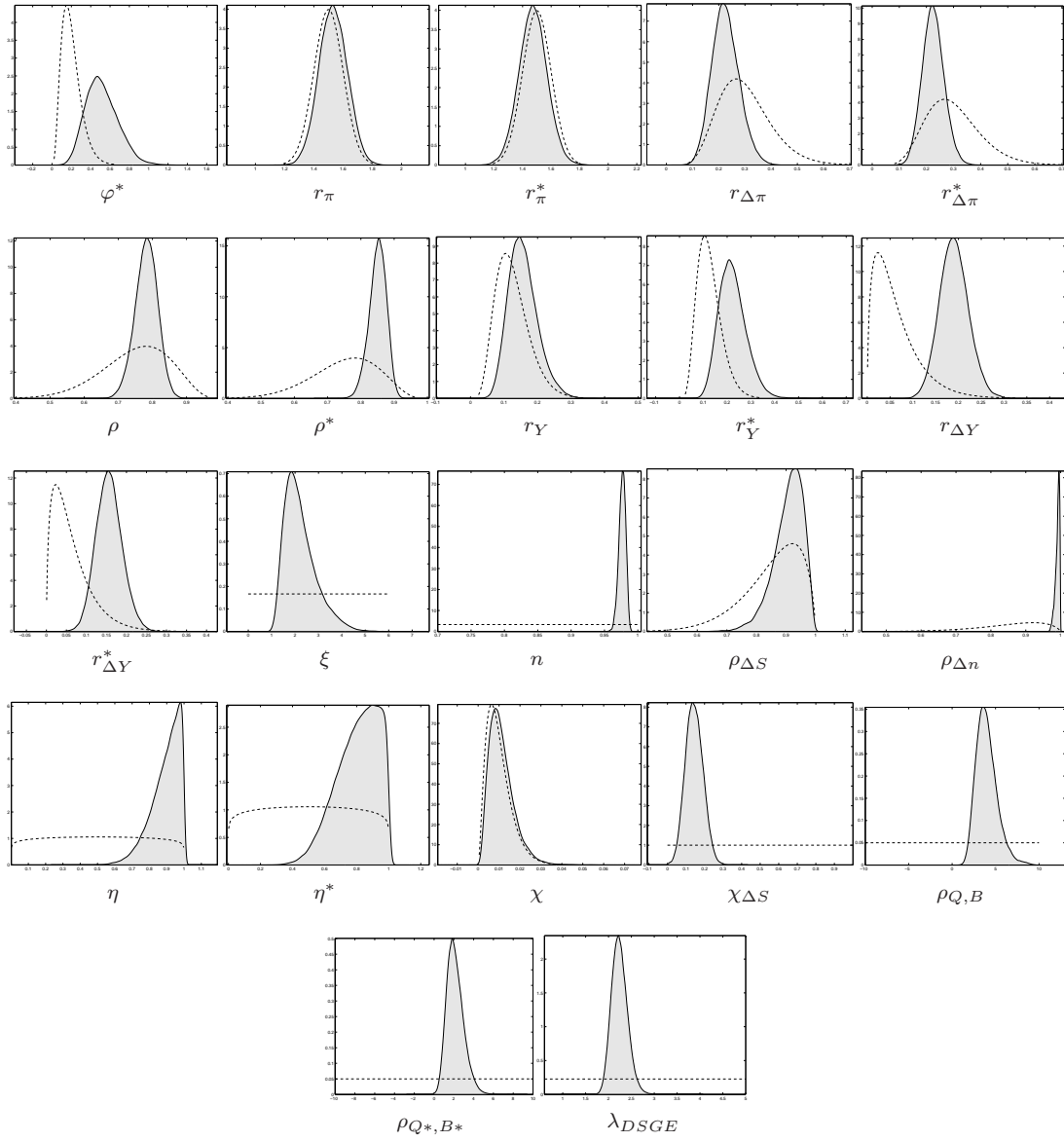


Fig. 3: Posterior densities.

Tab. 1: DSGE-VAR: PARAMETER ESTIMATES 1

| Shock names             | <i>A priori</i> beliefs |       |       | <i>A posteriori</i> beliefs |       |       |                 |                 |
|-------------------------|-------------------------|-------|-------|-----------------------------|-------|-------|-----------------|-----------------|
|                         | Distribution            | Mean  | Std.  | Median                      | Mean  | Std.  | $\mathcal{I}_1$ | $\mathcal{I}_2$ |
| $\epsilon_t^A$          | Uniform                 | 3.000 | 1.732 | 0.315                       | 0.316 | 0.025 | 0.276           | 0.358           |
| $\epsilon_t^B$          | Uniform                 | 5.000 | 2.887 | 0.924                       | 0.945 | 0.206 | 0.623           | 1.277           |
| $\epsilon_t^G$          | Uniform                 | 3.000 | 1.732 | 2.272                       | 2.281 | 0.191 | 1.974           | 2.598           |
| $\epsilon_t^L$          | Uniform                 | 3.000 | 1.732 | 0.237                       | 0.238 | 0.027 | 0.193           | 0.283           |
| $\epsilon_t^I$          | Uniform                 | 5.000 | 2.887 | 3.327                       | 3.359 | 0.577 | 2.400           | 4.269           |
| $\epsilon_t^R$          | Uniform                 | 3.000 | 1.732 | 0.163                       | 0.164 | 0.014 | 0.142           | 0.187           |
| $\epsilon_t^P$          | Uniform                 | 3.000 | 1.732 | 0.205                       | 0.206 | 0.019 | 0.174           | 0.235           |
| $\epsilon_t^{CPI}$      | Uniform                 | 3.000 | 1.732 | 0.153                       | 0.154 | 0.012 | 0.134           | 0.173           |
| $\epsilon_t^{A*}$       | Uniform                 | 3.000 | 1.732 | 0.449                       | 0.454 | 0.059 | 0.358           | 0.546           |
| $\epsilon_t^{B*}$       | Uniform                 | 5.000 | 2.887 | 0.942                       | 0.971 | 0.220 | 0.631           | 1.312           |
| $\epsilon_t^{G*}$       | Uniform                 | 3.000 | 1.732 | 1.716                       | 1.723 | 0.168 | 1.442           | 1.996           |
| $\epsilon_t^{L*}$       | Uniform                 | 3.000 | 1.732 | 0.085                       | 0.087 | 0.018 | 0.058           | 0.116           |
| $\epsilon_t^{I*}$       | Uniform                 | 5.000 | 2.887 | 3.771                       | 3.800 | 0.565 | 2.861           | 4.706           |
| $\epsilon_t^{R*}$       | Uniform                 | 3.000 | 1.732 | 0.108                       | 0.109 | 0.010 | 0.094           | 0.125           |
| $\epsilon_t^{P*}$       | Uniform                 | 3.000 | 1.732 | 0.224                       | 0.224 | 0.019 | 0.193           | 0.255           |
| $\epsilon_t^{CPI*}$     | Uniform                 | 3.000 | 1.732 | 0.197                       | 0.197 | 0.015 | 0.172           | 0.223           |
| $\epsilon_t^{\Delta S}$ | Uniform                 | 3.000 | 1.732 | 0.197                       | 0.209 | 0.077 | 0.089           | 0.323           |
| $\epsilon_t^{\Delta n}$ | Uniform                 | 3.000 | 1.732 | 0.353                       | 0.360 | 0.051 | 0.279           | 0.439           |



Tab. 2: DSGE-VAR: PARAMETER ESTIMATES 2

| Parameters            | <i>A priori</i> beliefs |       |       | <i>A posteriori</i> beliefs |        |       |                 |                 |
|-----------------------|-------------------------|-------|-------|-----------------------------|--------|-------|-----------------|-----------------|
|                       | Distribution            | Mean  | Std.  | Median                      | Mean   | Std.  | $\mathcal{I}_1$ | $\mathcal{I}_2$ |
| $\rho_A$              | Beta                    | 0.850 | 0.100 | 0.866                       | 0.863  | 0.035 | 0.808           | 0.919           |
| $\rho_B$              | Beta                    | 0.850 | 0.100 | 0.363                       | 0.367  | 0.091 | 0.216           | 0.515           |
| $\rho_G$              | Beta                    | 0.500 | 0.280 | 0.847                       | 0.815  | 0.119 | 0.627           | 0.973           |
| $\rho_L$              | Beta                    | 0.850 | 0.100 | 0.260                       | 0.264  | 0.071 | 0.148           | 0.378           |
| $\rho_I$              | Beta                    | 0.850 | 0.100 | 0.441                       | 0.441  | 0.085 | 0.302           | 0.580           |
| $\rho_{A^*}$          | Beta                    | 0.850 | 0.100 | 0.884                       | 0.882  | 0.030 | 0.833           | 0.930           |
| $\rho_{B^*}$          | Beta                    | 0.850 | 0.100 | 0.597                       | 0.588  | 0.117 | 0.401           | 0.781           |
| $\rho_{G^*}$          | Beta                    | 0.500 | 0.280 | 0.914                       | 0.909  | 0.034 | 0.857           | 0.960           |
| $\rho_{L^*}$          | Beta                    | 0.850 | 0.100 | 0.708                       | 0.700  | 0.092 | 0.556           | 0.854           |
| $\rho_{I^*}$          | Beta                    | 0.850 | 0.100 | 0.296                       | 0.299  | 0.071 | 0.181           | 0.413           |
| $\rho_{A,\Delta S}$   | Uniform                 | 0.000 | 1.732 | -0.275                      | -0.284 | 0.121 | -0.472          | -0.084          |
| $\rho_{G,\Delta S}$   | Uniform                 | 0.000 | 1.732 | 0.050                       | 0.062  | 0.042 | 0.009           | 0.124           |
| $\rho_{\Delta n,G}$   | Uniform                 | 0.000 | 3.464 | 3.214                       | 3.205  | 0.854 | 1.778           | 4.605           |
| $\rho_{G^*,\Delta S}$ | Uniform                 | 0.000 | 1.732 | -0.082                      | -0.086 | 0.028 | -0.129          | -0.044          |
| $\rho_{\Delta S}$     | Beta                    | 0.850 | 0.100 | 0.914                       | 0.907  | 0.050 | 0.834           | 0.985           |
| $\rho_{\Delta n}$     | Beta                    | 0.850 | 0.100 | 0.992                       | 0.991  | 0.006 | 0.982           | 1.000           |
| $\rho_{Q,B}$          | Uniform                 | 0.000 | 5.774 | 3.824                       | 3.963  | 1.209 | 2.017           | 5.794           |
| $\rho_{Q^*,B^*}$      | Uniform                 | 0.000 | 5.774 | 2.036                       | 2.132  | 0.854 | 0.739           | 3.456           |

Tab. 3: DSGE-VAR: PARAMETER ESTIMATES 3

| Parameters        | <i>A priori</i> beliefs |       |       | <i>A posteriori</i> beliefs |       |       |                 |                 |
|-------------------|-------------------------|-------|-------|-----------------------------|-------|-------|-----------------|-----------------|
|                   | Distribution            | Mean  | Std.  | Median                      | Mean  | Std.  | $\mathcal{I}_1$ | $\mathcal{I}_2$ |
| $\sigma_C$        | Normal                  | 1.000 | 0.375 | 0.763                       | 0.777 | 0.227 | 0.411           | 1.151           |
| $\sigma_C^*$      | Normal                  | 1.000 | 0.375 | 1.063                       | 1.079 | 0.271 | 0.635           | 1.521           |
| $h$               | Beta                    | 0.700 | 0.100 | 0.639                       | 0.635 | 0.072 | 0.518           | 0.757           |
| $h^*$             | Beta                    | 0.700 | 0.100 | 0.516                       | 0.518 | 0.106 | 0.346           | 0.695           |
| $\sigma_L$        | Gamma                   | 2.000 | 0.750 | 1.136                       | 1.260 | 0.654 | 0.293           | 2.204           |
| $\sigma_L^*$      | Gamma                   | 2.000 | 0.750 | 1.865                       | 1.960 | 0.753 | 0.757           | 3.076           |
| $\phi$            | Normal                  | 4.000 | 0.500 | 4.269                       | 4.274 | 0.475 | 3.497           | 5.058           |
| $\phi^*$          | Normal                  | 4.000 | 0.500 | 4.236                       | 4.236 | 0.465 | 3.472           | 4.992           |
| $\varphi$         | Gamma                   | 0.200 | 0.100 | 0.186                       | 0.200 | 0.088 | 0.062           | 0.331           |
| $\varphi^*$       | Gamma                   | 0.200 | 0.100 | 0.499                       | 0.514 | 0.166 | 0.246           | 0.777           |
| $\alpha_w$        | Beta                    | 0.750 | 0.050 | 0.746                       | 0.744 | 0.046 | 0.667           | 0.818           |
| $\alpha_w^*$      | Beta                    | 0.750 | 0.050 | 0.792                       | 0.789 | 0.045 | 0.715           | 0.861           |
| $\xi_w$           | Beta                    | 0.500 | 0.150 | 0.307                       | 0.319 | 0.118 | 0.130           | 0.511           |
| $\xi_w^*$         | Beta                    | 0.500 | 0.150 | 0.274                       | 0.285 | 0.108 | 0.110           | 0.454           |
| $\lambda_e$       | Beta                    | 0.750 | 0.050 | 0.783                       | 0.783 | 0.019 | 0.752           | 0.813           |
| $\alpha_H$        | Beta                    | 0.750 | 0.050 | 0.870                       | 0.869 | 0.023 | 0.830           | 0.907           |
| $\alpha_F^*$      | Beta                    | 0.750 | 0.050 | 0.907                       | 0.906 | 0.014 | 0.884           | 0.929           |
| $\gamma_H$        | Beta                    | 0.500 | 0.150 | 0.486                       | 0.486 | 0.106 | 0.316           | 0.665           |
| $\gamma_F^*$      | Beta                    | 0.500 | 0.150 | 0.287                       | 0.290 | 0.081 | 0.155           | 0.420           |
| $\eta$            | Beta                    | 0.500 | 0.280 | 0.911                       | 0.893 | 0.083 | 0.775           | 1.000           |
| $\eta^*$          | Beta                    | 0.500 | 0.280 | 0.822                       | 0.805 | 0.129 | 0.622           | 1.000           |
| $\xi$             | Uniform                 | 3.000 | 1.732 | 2.084                       | 2.220 | 0.680 | 1.218           | 3.251           |
| $n$               | Uniform                 | 0.850 | 0.087 | 0.977                       | 0.977 | 0.005 | 0.968           | 0.985           |
| $\chi$            | Gamma                   | 0.010 | 0.006 | 0.010                       | 0.011 | 0.006 | 0.002           | 0.020           |
| $\chi_{\Delta S}$ | Uniform                 | 0.500 | 0.289 | 0.142                       | 0.144 | 0.047 | 0.067           | 0.220           |
| $\rho$            | Beta                    | 0.750 | 0.100 | 0.783                       | 0.782 | 0.033 | 0.728           | 0.836           |
| $\rho^*$          | Beta                    | 0.750 | 0.100 | 0.852                       | 0.850 | 0.026 | 0.808           | 0.892           |
| $r_\pi$           | Normal                  | 1.500 | 0.100 | 1.529                       | 1.530 | 0.096 | 1.377           | 1.692           |
| $r_\pi^*$         | Normal                  | 1.500 | 0.100 | 1.472                       | 1.472 | 0.098 | 1.310           | 1.633           |
| $r_{\Delta\pi}$   | Gamma                   | 0.300 | 0.100 | 0.223                       | 0.226 | 0.051 | 0.141           | 0.310           |
| $r_{\Delta\pi}^*$ | Gamma                   | 0.300 | 0.100 | 0.223                       | 0.224 | 0.040 | 0.158           | 0.290           |
| $r_Y$             | Gamma                   | 0.125 | 0.050 | 0.152                       | 0.156 | 0.043 | 0.085           | 0.225           |
| $r_Y^*$           | Gamma                   | 0.125 | 0.050 | 0.217                       | 0.222 | 0.057 | 0.131           | 0.313           |
| $r_{\Delta Y}$    | Gamma                   | 0.063 | 0.050 | 0.189                       | 0.190 | 0.032 | 0.138           | 0.242           |
| $r_{\Delta Y}^*$  | Gamma                   | 0.063 | 0.050 | 0.155                       | 0.156 | 0.033 | 0.103           | 0.209           |
| $\lambda_{DSGE}$  | Uniform                 | 2.800 | 1.270 | 2.228                       | 2.238 | 0.174 | 1.948           | 2.511           |

Tab. 4: DIRECT ESTIMATIONS: POSTERIOR PARAMETERS 1

| Parameters              | augmented DSGE |                 |                 | DSGE |                 |                 |
|-------------------------|----------------|-----------------|-----------------|------|-----------------|-----------------|
|                         | Mean           | $\mathcal{I}_1$ | $\mathcal{I}_2$ | Mean | $\mathcal{I}_1$ | $\mathcal{I}_2$ |
| $\epsilon_t^A$          | 0.38           | 0.31            | 0.45            | 0.44 | 0.40            | 0.48            |
| $\epsilon_t^B$          | 1.85           | 1.24            | 2.35            | 1.72 | 1.27            | 2.22            |
| $\epsilon_t^G$          | 2.81           | 2.28            | 3.39            | 2.86 | 2.32            | 3.40            |
| $\epsilon_t^L$          | 0.01           | 0.00            | 0.02            | 0.33 | 0.28            | 0.38            |
| $\epsilon_t^I$          | 1.89           | 0.38            | 3.48            | 3.99 | 3.09            | 5.00            |
| $\epsilon_t^R$          | 0.23           | 0.20            | 0.25            | 0.24 | 0.22            | 0.27            |
| $\epsilon_t^P$          | 0.27           | 0.23            | 0.31            | 0.33 | 0.29            | 0.37            |
| $\epsilon_t^{CPI}$      | 0.15           | 0.12            | 0.18            | 0.23 | 0.21            | 0.25            |
| $\epsilon_t^{A*}$       | 1.04           | 0.71            | 1.37            | 1.08 | 0.79            | 1.37            |
| $\epsilon_t^{B*}$       | 1.75           | 1.31            | 2.16            | 1.71 | 1.16            | 2.17            |
| $\epsilon_t^{G*}$       | 2.00           | 1.46            | 2.46            | 1.95 | 1.59            | 2.30            |
| $\epsilon_t^{L*}$       | 0.03           | 0.02            | 0.04            | 0.10 | 0.07            | 0.13            |
| $\epsilon_t^{I*}$       | 0.66           | 0.26            | 1.11            | 5.63 | 4.38            | 6.89            |
| $\epsilon_t^{R*}$       | 0.11           | 0.08            | 0.14            | 0.16 | 0.14            | 0.18            |
| $\epsilon_t^{P*}$       | 0.33           | 0.29            | 0.37            | 0.36 | 0.32            | 0.40            |
| $\epsilon_t^{CPI*}$     | 0.26           | 0.23            | 0.29            | 0.30 | 0.27            | 0.33            |
| $\epsilon_t^{\Delta S}$ | 0.18           | 0.05            | 0.31            | 0.26 | 0.13            | 0.37            |
| $\epsilon_t^{\Delta n}$ | 0.55           | 0.38            | 0.74            | 0.42 | 0.24            | 0.59            |
| $\epsilon_t^Q$          | 5.88           | 3.57            | 8.34            |      |                 |                 |
| $\epsilon_t^{Q*}$       | 6.74           | 5.46            | 7.97            |      |                 |                 |
| $\epsilon_t^W$          | 0.41           | 0.36            | 0.45            |      |                 |                 |
| $\epsilon_t^{W*}$       | 0.23           | 0.19            | 0.27            |      |                 |                 |
| $f_t^A$                 | 0.23           | 0.13            | 0.31            |      |                 |                 |
| $f_t^I$                 | 0.42           | 0.14            | 0.70            |      |                 |                 |
| $f_t^R$                 | 0.12           | 0.09            | 0.14            |      |                 |                 |
| $f_t^{CPI}$             | 0.16           | 0.13            | 0.19            |      |                 |                 |

Tab. 5: DIRECT ESTIMATIONS: POSTERIOR PARAMETERS 2

| Parameters            | augmented DSGE |                 |                 | DSGE  |                 |                 |
|-----------------------|----------------|-----------------|-----------------|-------|-----------------|-----------------|
|                       | Mean           | $\mathcal{I}_1$ | $\mathcal{I}_2$ | Mean  | $\mathcal{I}_1$ | $\mathcal{I}_2$ |
| $\rho_{FA}$           | 0.91           | 0.84            | 0.98            |       |                 |                 |
| $\rho_{FI}$           | 0.87           | 0.77            | 0.97            |       |                 |                 |
| $\rho_{FR}$           | 0.48           | 0.35            | 0.61            |       |                 |                 |
| $\rho_{FCPI}$         | 0.56           | 0.41            | 0.71            |       |                 |                 |
| $\rho_A$              | 0.94           | 0.91            | 0.98            | 0.93  | 0.90            | 0.96            |
| $\rho_B$              | 0.36           | 0.21            | 0.53            | 0.39  | 0.23            | 0.54            |
| $\rho_G$              | 0.95           | 0.90            | 0.98            | 0.97  | 0.94            | 1.00            |
| $\rho_L$              | 0.98           | 0.95            | 1.00            | 0.28  | 0.15            | 0.39            |
| $\rho_I$              | 0.77           | 0.62            | 0.93            | 0.68  | 0.58            | 0.77            |
| $\rho_{A^*}$          | 0.92           | 0.88            | 0.97            | 0.89  | 0.85            | 0.93            |
| $\rho_{B^*}$          | 0.81           | 0.73            | 0.90            | 0.67  | 0.48            | 0.85            |
| $\rho_{G^*}$          | 0.90           | 0.81            | 0.98            | 0.88  | 0.78            | 0.97            |
| $\rho_{L^*}$          | 0.91           | 0.88            | 0.95            | 0.78  | 0.68            | 0.87            |
| $\rho_{I^*}$          | 0.90           | 0.81            | 0.97            | 0.40  | 0.25            | 0.53            |
| $\rho_{A,\Delta S}$   | -0.26          | -0.44           | -0.06           | -0.18 | -0.31           | -0.05           |
| $\rho_{G,\Delta S}$   | 0.03           | -0.01           | 0.09            | 0.03  | 0.00            | 0.05            |
| $\rho_{\Delta n,G^*}$ | 1.64           | 0.79            | 2.65            | 1.75  | 0.72            | 2.62            |
| $\rho_{G^*,\Delta S}$ | -0.09          | -0.19           | 0.04            | -0.14 | -0.25           | -0.02           |
| $\rho_{\Delta S}$     | 0.91           | 0.82            | 0.99            | 0.93  | 0.88            | 0.99            |
| $\rho_{\Delta n}$     | 0.99           | 0.97            | 1.00            | 0.98  | 0.95            | 1.00            |
| $\rho_{Q,B}$          | 2.61           | 1.45            | 3.86            | 2.38  | 1.44            | 3.26            |
| $\rho_{Q,B}$          | 2.14           | 1.30            | 3.01            | 2.46  | 1.47            | 3.37            |

Tab. 6: DIRECT ESTIMATIONS: POSTERIOR PARAMETERS 3

| Parameters        | augmented DSGE |                 |                 | DSGE |                 |                 |
|-------------------|----------------|-----------------|-----------------|------|-----------------|-----------------|
|                   | Mean           | $\mathcal{I}_1$ | $\mathcal{I}_2$ | Mean | $\mathcal{I}_1$ | $\mathcal{I}_2$ |
| $\sigma_C$        | 0.75           | 0.43            | 1.11            | 0.67 | 0.36            | 0.99            |
| $\sigma_C^*$      | 1.23           | 0.74            | 1.67            | 0.88 | 0.33            | 1.42            |
| $h$               | 0.75           | 0.67            | 0.84            | 0.76 | 0.68            | 0.85            |
| $h^*$             | 0.52           | 0.38            | 0.65            | 0.64 | 0.44            | 0.83            |
| $\sigma_L$        | 2.10           | 0.93            | 3.23            | 1.69 | 0.53            | 2.70            |
| $\sigma_L^*$      | 1.65           | 0.59            | 2.67            | 1.88 | 0.79            | 2.96            |
| $\phi$            | 4.70           | 4.00            | 5.40            | 4.62 | 3.93            | 5.36            |
| $\phi^*$          | 5.08           | 4.39            | 5.74            | 4.99 | 4.28            | 5.72            |
| $\varphi$         | 0.31           | 0.12            | 0.50            | 0.27 | 0.11            | 0.42            |
| $\varphi^*$       | 0.26           | 0.07            | 0.44            | 0.21 | 0.08            | 0.34            |
| $\alpha_w$        | 0.79           | 0.74            | 0.85            | 0.83 | 0.78            | 0.87            |
| $\alpha_w^*$      | 0.86           | 0.81            | 0.90            | 0.84 | 0.79            | 0.90            |
| $\xi_w$           | 0.40           | 0.16            | 0.59            | 0.49 | 0.28            | 0.69            |
| $\xi_w^*$         | 0.22           | 0.08            | 0.36            | 0.20 | 0.06            | 0.33            |
| $\lambda_e$       | 0.88           | 0.85            | 0.90            | 0.87 | 0.85            | 0.89            |
| $\alpha_H$        | 0.84           | 0.81            | 0.87            | 0.84 | 0.81            | 0.87            |
| $\alpha_F^*$      | 0.93           | 0.91            | 0.94            | 0.91 | 0.90            | 0.93            |
| $\gamma_H$        | 0.46           | 0.27            | 0.66            | 0.76 | 0.66            | 0.86            |
| $\gamma_F^*$      | 0.35           | 0.23            | 0.47            | 0.42 | 0.30            | 0.54            |
| $\eta$            | 0.89           | 0.77            | 1.00            | 0.87 | 0.73            | 1.00            |
| $\eta^*$          | 0.68           | 0.45            | 0.91            | 0.73 | 0.53            | 0.96            |
| $\xi$             | 2.51           | 1.36            | 3.66            | 2.26 | 1.20            | 3.52            |
| $n$               | 0.97           | 0.97            | 0.98            | 0.98 | 0.97            | 0.99            |
| $\chi$            | 0.02           | 0.01            | 0.03            | 0.02 | 0.01            | 0.03            |
| $\chi_{\Delta S}$ | 0.11           | 0.04            | 0.17            | 0.15 | 0.06            | 0.23            |
| $\rho$            | 0.78           | 0.74            | 0.82            | 0.82 | 0.78            | 0.85            |
| $\rho^*$          | 0.86           | 0.82            | 0.89            | 0.89 | 0.87            | 0.92            |
| $r_\pi$           | 1.58           | 1.43            | 1.71            | 1.59 | 1.43            | 1.73            |
| $r_\pi^*$         | 1.46           | 1.30            | 1.64            | 1.57 | 1.42            | 1.72            |
| $r_{\Delta\pi}$   | 0.27           | 0.17            | 0.35            | 0.25 | 0.17            | 0.33            |
| $r_{\Delta\pi}^*$ | 0.18           | 0.13            | 0.24            | 0.18 | 0.12            | 0.23            |
| $r_Y$             | 0.07           | 0.04            | 0.11            | 0.06 | 0.03            | 0.10            |
| $r_Y^*$           | 0.13           | 0.06            | 0.20            | 0.16 | 0.08            | 0.22            |
| $r_{\Delta Y}$    | 0.19           | 0.15            | 0.23            | 0.18 | 0.14            | 0.22            |
| $r_{\Delta Y}^*$  | 0.20           | 0.16            | 0.24            | 0.19 | 0.16            | 0.23            |

Tab. 7: COMPARISON CONDITIONAL AND UNCONDITIONAL MOMENTS

|                           | DSGE-VAR        |       |       |          | DSGE            |       |       |          | Data          |       |       |       |
|---------------------------|-----------------|-------|-------|----------|-----------------|-------|-------|----------|---------------|-------|-------|-------|
|                           | <i>quarters</i> |       |       |          | <i>quarters</i> |       |       |          | <i>sample</i> |       |       |       |
|                           | 4               | 12    | 20    | $\infty$ | 4               | 12    | 20    | $\infty$ | 70-05         | 73-05 | 76-05 | 85-05 |
| <u>Standard deviation</u> |                 |       |       |          |                 |       |       |          |               |       |       |       |
| US variables              |                 |       |       |          |                 |       |       |          |               |       |       |       |
| $Z_t$                     | 1.15            | 1.58  | 1.70  | 1.84     | 1.13            | 1.32  | 1.35  | 1.37     | 2.83          | 2.74  | 2.58  | 1.78  |
| $C_t$                     | 0.89            | 1.45  | 1.66  | 2.14     | 0.73            | 0.88  | 0.97  | 1.83     | 2.93          | 2.78  | 2.67  | 1.97  |
| $I_t$                     | 3.11            | 4.81  | 5.28  | 5.66     | 2.85            | 3.95  | 4.12  | 4.51     | 7.32          | 7.54  | 7.58  | 7.55  |
| $L_t$                     | 1.20            | 1.65  | 1.84  | 2.36     | 1.02            | 1.15  | 1.15  | 1.19     | 3.30          | 3.34  | 3.29  | 3.19  |
| $w_t$                     | 1.04            | 1.35  | 1.52  | 1.76     | 1.01            | 1.15  | 1.16  | 1.21     | 2.30          | 2.36  | 2.42  | 2.81  |
| $\Pi_t$                   | 0.30            | 0.34  | 0.35  | 0.36     | 0.28            | 0.30  | 0.30  | 0.31     | 0.49          | 0.46  | 0.43  | 0.30  |
| $\Pi_{H,t}$               | 0.26            | 0.29  | 0.31  | 0.33     | 0.24            | 0.26  | 0.26  | 0.27     | 0.45          | 0.43  | 0.40  | 0.29  |
| $R_t$                     | 0.32            | 0.39  | 0.41  | 0.43     | 0.26            | 0.28  | 0.28  | 0.28     | 0.70          | 0.65  | 0.66  | 0.40  |
| euro area variables       |                 |       |       |          |                 |       |       |          |               |       |       |       |
| $Z_t^*$                   | 0.88            | 1.21  | 1.35  | 1.50     | 0.84            | 1.00  | 1.06  | 1.07     | 2.33          | 2.26  | 2.26  | 1.89  |
| $C_t^*$                   | 0.86            | 1.37  | 1.64  | 2.09     | 0.78            | 1.02  | 1.15  | 1.92     | 2.56          | 2.34  | 2.44  | 1.75  |
| $I_t^*$                   | 2.17            | 3.35  | 3.98  | 4.47     | 2.14            | 2.99  | 3.30  | 3.67     | 6.07          | 6.28  | 6.26  | 6.14  |
| $L_t^*$                   | 0.52            | 1.21  | 1.56  | 1.84     | 0.50            | 0.85  | 0.93  | 1.00     | 3.06          | 2.94  | 3.05  | 3.18  |
| $w_t^*$                   | 1.27            | 2.27  | 2.53  | 2.65     | 1.28            | 1.98  | 2.04  | 2.08     | 5.34          | 3.36  | 3.39  | 2.96  |
| $\Pi_t^*$                 | 0.34            | 0.38  | 0.40  | 0.41     | 0.34            | 0.37  | 0.37  | 0.37     | 0.56          | 0.49  | 0.47  | 0.34  |
| $\Pi_{F,t}^*$             | 0.30            | 0.34  | 0.35  | 0.37     | 0.28            | 0.31  | 0.32  | 0.32     | 0.52          | 0.44  | 0.42  | 0.31  |
| $R_t^*$                   | 0.24            | 0.34  | 0.37  | 0.38     | 0.20            | 0.25  | 0.25  | 0.26     | 0.65          | 0.59  | 0.58  | 0.51  |
| $\Delta S_t$              | 3.67            | 3.78  | 3.80  | 3.83     | 3.35            | 3.41  | 3.42  | 3.42     | 4.51          | 4.67  | 4.60  | 4.72  |
| $CA_t$                    | 0.52            | 0.74  | 0.82  | 0.96     | 0.48            | 0.59  | 0.61  | 0.63     | 1.13          | 1.15  | 1.19  | 1.34  |
| <u>Correlations</u>       |                 |       |       |          |                 |       |       |          |               |       |       |       |
| $Z_t, Z_t^*$              | 0.12            | 0.23  | 0.25  | 0.27     | 0.09            | 0.05  | 0.04  | 0.03     | 0.29          | 0.44  | 0.41  | 0.43  |
| $C_t, C_t^*$              | 0.03            | 0.07  | -0.04 | -0.33    | 0.00            | -0.08 | -0.17 | -0.65    | 0.00          | 0.18  | 0.20  | 0.22  |
| $I_t, I_t^*$              | 0.00            | 0.09  | 0.10  | 0.02     | -0.02           | 0.01  | 0.01  | -0.14    | 0.12          | 0.12  | 0.09  | -0.05 |
| $L_t, L_t^*$              | 0.05            | 0.10  | 0.00  | -0.11    | 0.06            | 0.03  | 0.03  | -0.02    | 0.11          | 0.04  | 0.02  | -0.20 |
| $w_t, w_t^*$              | -0.03           | -0.15 | -0.24 | -0.30    | 0.00            | 0.00  | 0.00  | -0.04    | -0.33         | -0.39 | -0.35 | -0.51 |
| $\Pi_t, \Pi_t^*$          | 0.16            | 0.22  | 0.26  | 0.29     | 0.00            | 0.00  | 0.00  | -0.03    | 0.61          | 0.55  | 0.48  | 0.53  |
| $\Pi_{H,t}, \Pi_{F,t}^*$  | 0.09            | 0.18  | 0.23  | 0.27     | -0.02           | -0.02 | -0.02 | -0.04    | 0.72          | 0.65  | 0.58  | 0.59  |
| $R_t, R_t^*$              | 0.42            | 0.43  | 0.42  | 0.43     | 0.16            | 0.13  | 0.12  | 0.09     | 0.70          | 0.64  | 0.59  | 0.20  |
| $\Delta S_t, CA_t$        | -0.11           | -0.19 | -0.21 | -0.22    | -0.13           | -0.20 | -0.21 | -0.20    | -0.25         | -0.24 | -0.25 | -0.28 |

Tab. 8: COMPARISON OF THE SHOCK DECOMPOSITION OF UNCONDITIONAL VARIANCES

|                            | DSGE-VAR |         |       |              |         |           |         | DSGE  |         |       |              |         |           |         |
|----------------------------|----------|---------|-------|--------------|---------|-----------|---------|-------|---------|-------|--------------|---------|-----------|---------|
|                            | $Z_t$    | $\Pi_t$ | $R_t$ | $\Delta S_t$ | $Z_t^*$ | $\Pi_t^*$ | $R_t^*$ | $Z_t$ | $\Pi_t$ | $R_t$ | $\Delta S_t$ | $Z_t^*$ | $\Pi_t^*$ | $R_t^*$ |
| <u>US shocks</u>           | 61.1     | 71.3    | 72.8  | 13.0         | 14.6    | 8.2       | 12.2    | 86.3  | 89.1    | 86.9  | 13.4         | 1.8     | 0.5       | 1.8     |
| $\varepsilon^A$            | 1.9      | 5.2     | 6.0   | 1.5          | 0.7     | 0.4       | 0.7     | 6.3   | 9.5     | 4.6   | 3.1          | 0.4     | 0.1       | 0.5     |
| $\varepsilon^L$            | 4.4      | 8.6     | 1.9   | 0.8          | 1.4     | 0.2       | 1.3     | 6.8   | 13.2    | 8.6   | 0.1          | 0.0     | 0.0       | 0.0     |
| $\varepsilon^I$            | 6.1      | 1.5     | 19.7  | 0.8          | 2.6     | 0.6       | 1.2     | 12.2  | 0.4     | 10.3  | 0.5          | 0.1     | 0.0       | 0.1     |
| $\varepsilon^B$            | 16.4     | 2.0     | 18.5  | 1.9          | 1.6     | 1.3       | 3.3     | 23.2  | 0.3     | 19.5  | 0.7          | 0.2     | 0.0       | 0.1     |
| $\varepsilon^G$            | 5.5      | 0.4     | 3.0   | 5.3          | 1.1     | 1.3       | 1.9     | 12.0  | 0.3     | 8.0   | 7.2          | 1.0     | 0.3       | 0.9     |
| $\varepsilon^P$            | 9.7      | 31.3    | 5.8   | 0.8          | 1.5     | 1.2       | 1.3     | 8.5   | 41.4    | 4.1   | 0.1          | 0.0     | 0.0       | 0.0     |
| $\varepsilon^{CPI}$        | 1.8      | 18.6    | 1.5   | 0.3          | 0.6     | 1.0       | 0.5     | 1.6   | 22.3    | 1.2   | 0.1          | 0.0     | 0.0       | 0.0     |
| $\varepsilon^R$            | 15.2     | 3.6     | 16.4  | 1.6          | 5.0     | 2.2       | 2.2     | 15.6  | 1.7     | 30.5  | 1.6          | 0.1     | 0.1       | 0.1     |
| <u>EA shocks</u>           | 29.8     | 23.5    | 20.7  | 23.1         | 82.2    | 84.1      | 83.1    | 1.5   | 1.1     | 3.2   | 18.6         | 94.7    | 94.9      | 90.3    |
| $\varepsilon^{A^*}$        | 1.0      | 0.7     | 2.7   | 0.7          | 5.0     | 4.4       | 4.0     | 0.0   | 0.0     | 0.0   | 0.1          | 6.4     | 8.6       | 12.2    |
| $\varepsilon^{L^*}$        | 5.7      | 3.2     | 3.5   | 0.8          | 10.1    | 25.4      | 28.8    | 0.0   | 0.0     | 0.1   | 0.3          | 17.1    | 27.0      | 34.3    |
| $\varepsilon^{I^*}$        | 2.1      | 2.9     | 3.0   | 0.5          | 6.0     | 1.0       | 7.5     | 0.0   | 0.0     | 0.1   | 0.2          | 11.0    | 0.1       | 5.5     |
| $\varepsilon^{B^*}$        | 5.9      | 2.7     | 2.2   | 0.9          | 17.3    | 1.0       | 12.8    | 0.1   | 0.0     | 0.1   | 0.5          | 22.7    | 0.0       | 12.7    |
| $\varepsilon^{G^*}$        | 4.3      | 3.3     | 1.8   | 17.0         | 4.8     | 2.0       | 2.5     | 1.3   | 1.0     | 2.8   | 16.2         | 4.6     | 0.7       | 1.2     |
| $\varepsilon^{P^*}$        | 3.4      | 0.9     | 2.8   | 1.0          | 11.0    | 20.0      | 7.7     | 0.0   | 0.0     | 0.0   | 0.1          | 8.3     | 33.0      | 3.5     |
| $\varepsilon^{CPI^*}$      | 1.1      | 2.9     | 1.8   | 1.1          | 4.7     | 22.8      | 1.8     | 0.0   | 0.0     | 0.0   | 0.1          | 3.4     | 25.2      | 1.9     |
| $\varepsilon^{R^*}$        | 6.4      | 6.8     | 3.0   | 1.1          | 23.3    | 7.5       | 18.1    | 0.1   | 0.0     | 0.1   | 0.9          | 21.1    | 0.3       | 19.1    |
| <u>Open economy shocks</u> |          |         |       |              |         |           |         |       |         |       |              |         |           |         |
| $\varepsilon^{\Delta S}$   | 3.7      | 2.8     | 4.7   | 23.3         | 2.3     | 3.2       | 2.9     | 1.7   | 1.4     | 3.8   | 23.5         | 2.3     | 1.1       | 3.6     |
| $\varepsilon^{\Delta n}$   | 5.5      | 2.5     | 1.8   | 40.6         | 1.0     | 4.6       | 1.8     | 10.5  | 8.3     | 6.2   | 44.5         | 1.2     | 3.5       | 4.3     |

Tab. 9: COMPARISON OF THE SHOCK DECOMPOSITION OF CONDITIONAL COVARIANCE: US-EA GDP. The contributions of shocks sum to the conditional covariance.

| 10 <sup>-2</sup><br>quarters | DSGE-VAR |      |      |      |      |      |      | DSGE |      |      |      |      |      |      |
|------------------------------|----------|------|------|------|------|------|------|------|------|------|------|------|------|------|
|                              | 1        | 4    | 8    | 12   | 20   | 40   | ∞    | 1    | 4    | 8    | 12   | 20   | 40   | ∞    |
| <u>Cov. Cond.</u>            | 3.9      | 12.1 | 27.2 | 43.3 | 56.7 | 72.0 | 75.5 | 4.9  | 8.2  | 7.3  | 6.7  | 6.1  | 5.3  | 4.4  |
| <u>US shocks</u>             | 3.4      | 6.5  | 12.0 | 18.9 | 25.6 | 28.7 | 29.3 | 3.8  | 6.8  | 6.8  | 6.8  | 6.9  | 6.9  | 6.9  |
| $\varepsilon^A$              | 0.1      | 0.1  | -0.7 | -1.3 | -1.6 | -1.3 | -1.2 | 0.1  | -0.1 | -0.5 | -0.5 | -0.4 | -0.4 | -0.4 |
| $\varepsilon^L$              | 0.0      | 0.3  | 2.4  | 4.7  | 6.3  | 6.4  | 6.5  | 0.0  | 0.0  | -0.1 | -0.1 | -0.1 | -0.1 | -0.1 |
| $\varepsilon^I$              | 0.4      | 1.0  | 0.9  | 1.1  | 3.0  | 5.3  | 5.3  | 0.4  | 1.3  | 1.5  | 1.5  | 1.5  | 1.5  | 1.5  |
| $\varepsilon^B$              | 1.2      | 3.8  | 4.3  | 4.2  | 5.4  | 5.6  | 5.8  | 1.3  | 2.8  | 2.8  | 2.8  | 2.8  | 2.8  | 2.8  |
| $\varepsilon^G$              | 2.3      | 3.5  | 3.1  | 2.6  | 2.4  | 2.8  | 2.7  | 2.6  | 4.8  | 5.0  | 5.0  | 5.0  | 5.0  | 5.0  |
| $\varepsilon^P$              | -0.1     | 0.5  | 0.6  | 1.3  | 2.6  | 2.2  | 2.4  | -0.1 | -0.5 | -0.6 | -0.5 | -0.5 | -0.5 | -0.5 |
| $\varepsilon^{CPI}$          | -0.1     | -0.6 | 0.5  | 0.7  | 0.6  | 0.6  | 0.7  | -0.1 | -0.1 | -0.1 | -0.1 | -0.1 | -0.1 | -0.1 |
| $\varepsilon^R$              | -0.5     | -2.1 | 1.0  | 5.7  | 6.8  | 7.1  | 7.2  | -0.5 | -1.3 | -1.3 | -1.3 | -1.3 | -1.3 | -1.3 |
| <u>EA shocks</u>             | 2.1      | 7.3  | 15.9 | 25.8 | 32.7 | 43.1 | 45.8 | 2.4  | 4.7  | 4.8  | 4.8  | 4.9  | 4.9  | 4.9  |
| $\varepsilon^{A*}$           | 0.2      | 0.4  | 0.5  | 0.6  | 1.7  | 2.1  | 2.2  | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  |
| $\varepsilon^{L*}$           | 0.1      | 0.0  | 2.9  | 9.1  | 15.6 | 15.7 | 16.4 | 0.0  | 0.0  | -0.1 | -0.1 | -0.1 | 0.0  | 0.0  |
| $\varepsilon^{I*}$           | 0.2      | 0.6  | 0.2  | 0.5  | 1.1  | 1.7  | 2.1  | 0.3  | 0.9  | 1.0  | 1.0  | 1.0  | 1.0  | 1.0  |
| $\varepsilon^{B*}$           | 1.0      | 0.6  | -4.3 | -4.5 | -2.4 | -2.2 | -2.1 | 0.8  | 2.0  | 2.1  | 2.1  | 2.1  | 2.1  | 2.1  |
| $\varepsilon^{G*}$           | 0.9      | 0.8  | 1.3  | 1.9  | 1.1  | 4.2  | 4.2  | 1.7  | 3.3  | 3.5  | 3.5  | 3.5  | 3.5  | 3.5  |
| $\varepsilon^{P*}$           | 0.3      | 5.5  | 9.0  | 7.7  | 7.9  | 8.3  | 8.6  | -0.1 | -0.4 | -0.5 | -0.5 | -0.5 | -0.5 | -0.5 |
| $\varepsilon^{CPI*}$         | -0.1     | 0.0  | 2.5  | 3.7  | 3.3  | 3.4  | 3.5  | -0.1 | -0.2 | -0.2 | -0.2 | -0.2 | -0.2 | -0.2 |
| $\varepsilon^{R*}$           | -0.4     | -0.6 | 3.8  | 6.8  | 4.4  | 10.0 | 11.0 | -0.4 | -1.0 | -1.1 | -1.1 | -1.1 | -1.1 | -1.1 |
| <u>Open economy shocks</u>   |          |      |      |      |      |      |      |      |      |      |      |      |      |      |
| $\varepsilon^{\Delta S}$     | -1.4     | -2.6 | -2.0 | -2.5 | -1.9 | -1.2 | -0.9 | -1.2 | -2.7 | -2.9 | -2.9 | -2.9 | -2.9 | -2.9 |
| $\varepsilon^{\Delta n}$     | -0.2     | 0.9  | 1.4  | 1.1  | 0.4  | 1.4  | 1.3  | -0.2 | -0.6 | -1.5 | -2.0 | -2.8 | -3.6 | -4.5 |



Tab. 10: COMPARISON OF THE SHOCK DECOMPOSITION OF CONDITIONAL COVARIANCE: US-EA CONSUMPTION. The contributions of shocks sum to the conditional covariance.

| $10^{-2}$                  | DSGE-VAR |      |       |       |       |       |        | DSGE     |      |      |      |       |       |       |          |
|----------------------------|----------|------|-------|-------|-------|-------|--------|----------|------|------|------|-------|-------|-------|----------|
|                            | quarters | 1    | 4     | 8     | 12    | 20    | 40     | $\infty$ | 1    | 4    | 8    | 12    | 20    | 40    | $\infty$ |
| <u>Cov. Cond.</u>          | 0.3      | 2.3  | 12.5  | 12.9  | -11.0 | -73.2 | -147.2 |          | 0.0  | 0.2  | -2.0 | -6.8  | -19.4 | -55.7 | -228.2   |
| <u>US shocks</u>           | 0.1      | 2.9  | 12.0  | 17.8  | 19.1  | 1.6   | -18.1  |          | 0.0  | 0.9  | 1.3  | 1.2   | 1.0   | 0.9   | 0.2      |
| $\varepsilon^A$            | 0.0      | 0.1  | 0.0   | -0.4  | -0.5  | -0.5  | -0.5   |          | 0.0  | 0.2  | 0.5  | 0.5   | 0.3   | 0.0   | -0.5     |
| $\varepsilon^L$            | 0.0      | 0.7  | 2.2   | 2.9   | 2.9   | 2.7   | 0.9    |          | 0.0  | 0.1  | 0.2  | 0.2   | 0.1   | 0.1   | 0.1      |
| $\varepsilon^I$            | 0.1      | 0.6  | 2.3   | 3.2   | 4.6   | 4.3   | 4.2    |          | 0.0  | 0.2  | 0.3  | 0.3   | 0.5   | 0.9   | 1.2      |
| $\varepsilon^B$            | -0.3     | -1.9 | -2.5  | -2.8  | -3.5  | -10.7 | -14.8  |          | -0.2 | -0.7 | -0.7 | -0.7  | -0.7  | -0.6  | -0.6     |
| $\varepsilon^G$            | 0.2      | 1.5  | 2.8   | 2.3   | -0.1  | -9.2  | -21.6  |          | 0.1  | 0.7  | 0.8  | 0.7   | 0.5   | 0.2   | -0.2     |
| $\varepsilon^P$            | 0.0      | 0.8  | 1.0   | 2.2   | 4.6   | 4.3   | 4.2    |          | 0.0  | 0.1  | 0.1  | 0.1   | 0.1   | 0.1   | 0.1      |
| $\varepsilon^{CPI}$        | 0.0      | -0.1 | 0.9   | 0.9   | 0.9   | 0.8   | -0.2   |          | 0.0  | 0.0  | 0.0  | 0.0   | 0.0   | 0.0   | 0.0      |
| $\varepsilon^R$            | 0.1      | 1.1  | 5.4   | 9.5   | 10.2  | 9.8   | 9.6    |          | 0.1  | 0.2  | 0.2  | 0.2   | 0.1   | 0.1   | 0.1      |
| <u>EA shocks</u>           | 0.4      | 2.6  | 10.1  | 12.7  | 1.9   | -23.0 | -49.3  |          | 0.1  | 1.7  | 3.4  | 3.9   | 3.7   | 2.5   | -0.3     |
| $\varepsilon^{A^*}$        | 0.0      | -0.1 | -0.4  | -1.0  | -2.2  | -16.1 | -22.8  |          | 0.0  | 0.1  | 0.3  | 0.3   | 0.3   | 0.3   | 0.2      |
| $\varepsilon^{L^*}$        | 0.0      | 0.7  | 5.6   | 10.7  | 16.4  | 18.4  | 18.3   |          | 0.0  | 0.3  | 0.8  | 1.1   | 1.1   | 1.0   | 0.7      |
| $\varepsilon^{I^*}$        | 0.0      | 0.3  | 1.2   | 1.8   | 2.3   | 2.3   | 2.2    |          | 0.0  | 0.1  | 0.1  | 0.1   | 0.3   | 0.6   | 0.7      |
| $\varepsilon^{B^*}$        | -0.1     | -5.8 | -16.9 | -22.3 | -23.3 | -23.5 | -25.0  |          | -0.2 | -0.6 | -0.6 | -0.6  | -0.6  | -0.6  | -0.7     |
| $\varepsilon^{G^*}$        | 0.3      | 2.8  | 8.4   | 8.6   | -1.6  | -13.2 | -29.7  |          | 0.2  | 1.5  | 2.6  | 2.8   | 2.4   | 1.1   | -1.4     |
| $\varepsilon^{P^*}$        | 0.1      | 2.2  | 2.9   | 2.0   | 2.3   | 2.9   | 2.3    |          | 0.0  | 0.1  | 0.1  | 0.1   | 0.1   | 0.1   | 0.1      |
| $\varepsilon^{CPI^*}$      | 0.1      | 0.6  | 2.7   | 3.3   | 2.5   | 2.1   | 2.2    |          | 0.0  | 0.0  | 0.0  | 0.0   | 0.0   | 0.0   | 0.0      |
| $\varepsilon^{R^*}$        | 0.2      | 1.8  | 6.6   | 9.5   | 5.5   | 4.3   | 3.2    |          | 0.0  | 0.2  | 0.2  | 0.1   | 0.1   | 0.1   | 0.1      |
| <u>Open economy shocks</u> |          |      |       |       |       |       |        |          |      |      |      |       |       |       |          |
| $\varepsilon^{\Delta S}$   | -0.2     | -2.2 | -7.7  | -13.3 | -18.1 | -21.3 | -25.9  |          | -0.2 | -1.3 | -1.9 | -1.9  | -2.0  | -2.4  | -3.9     |
| $\varepsilon^{\Delta n}$   | 0.0      | -1.0 | -1.8  | -4.3  | -13.9 | -30.6 | -53.8  |          | 0.0  | -1.1 | -4.9 | -10.1 | -22.2 | -56.7 | -224.3   |

Tab. 11: COMPARISON OF THE SHOCK DECOMPOSITION OF CONDITIONAL COVARIANCE: US-EA CPI INFLATION RATE. The contributions of shocks sum to the conditional covariance.

| $10^{-3}$                  | DSGE-VAR |      |      |      |      |      |      | DSGE     |      |      |      |      |      |      |          |
|----------------------------|----------|------|------|------|------|------|------|----------|------|------|------|------|------|------|----------|
|                            | quarters | 1    | 4    | 8    | 12   | 20   | 40   | $\infty$ | 1    | 4    | 8    | 12   | 20   | 40   | $\infty$ |
| <u>Cov. Cond.</u>          | 5.9      | 16.4 | 23.2 | 28.1 | 36.4 | 43.2 | 43.6 |          | -2.4 | -1.9 | -1.9 | -2.2 | -2.6 | -3.3 | -4.7     |
| <u>US shocks</u>           | 1.4      | 6.1  | 7.4  | 8.5  | 10.2 | 11.9 | 11.9 |          | 0.7  | 1.1  | 1.1  | 1.1  | 1.1  | 1.1  | 1.1      |
| $\varepsilon^A$            | 0.4      | 1.6  | 1.6  | 1.6  | 1.6  | 1.6  | 1.7  |          | 0.3  | 0.4  | 0.4  | 0.4  | 0.4  | 0.4  | 0.4      |
| $\varepsilon^L$            | 0.1      | 1.0  | 1.2  | 1.2  | 1.2  | 1.2  | 1.2  |          | 0.1  | 0.2  | 0.2  | 0.2  | 0.2  | 0.2  | 0.2      |
| $\varepsilon^I$            | 0.1      | 0.3  | 0.6  | 0.6  | 0.6  | 1.0  | 1.1  |          | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0      |
| $\varepsilon^B$            | 0.0      | 0.5  | 1.1  | 1.2  | 1.0  | 1.2  | 1.2  |          | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0      |
| $\varepsilon^G$            | -0.1     | 0.0  | 0.2  | 0.4  | 0.4  | 0.4  | 0.3  |          | -0.3 | -0.3 | -0.3 | -0.3 | -0.3 | -0.3 | -0.3     |
| $\varepsilon^P$            | 0.7      | 2.1  | 2.0  | 1.9  | 2.1  | 2.6  | 2.6  |          | 0.5  | 0.6  | 0.6  | 0.6  | 0.6  | 0.6  | 0.6      |
| $\varepsilon^{CPI}$        | 0.4      | 1.0  | 1.3  | 1.6  | 1.7  | 1.8  | 1.8  |          | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3      |
| $\varepsilon^R$            | -0.2     | -0.5 | -0.6 | 0.0  | 1.5  | 2.0  | 2.0  |          | -0.1 | -0.1 | 0.0  | 0.0  | 0.0  | 0.0  | 0.0      |
| <u>EA shocks</u>           | 8.5      | 15.7 | 20.8 | 24.7 | 31.2 | 36.0 | 36.5 |          | 0.5  | 1.2  | 1.5  | 1.5  | 1.5  | 1.5  | 1.5      |
| $\varepsilon^{A^*}$        | 0.0      | -0.2 | -0.5 | -0.6 | -0.6 | -0.4 | -0.4 |          | 0.1  | 0.2  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3      |
| $\varepsilon^{L^*}$        | 2.3      | 7.4  | 10.8 | 11.2 | 11.2 | 12.2 | 12.4 |          | 0.2  | 0.6  | 0.8  | 0.9  | 0.9  | 0.9  | 0.9      |
| $\varepsilon^{I^*}$        | -0.1     | 0.4  | 0.4  | 0.5  | 0.9  | 1.5  | 1.6  |          | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0      |
| $\varepsilon^{B^*}$        | -0.1     | 0.3  | 0.7  | 0.8  | 0.9  | 1.3  | 1.3  |          | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0      |
| $\varepsilon^{G^*}$        | -0.5     | 0.1  | 0.2  | 0.7  | 1.5  | 1.7  | 1.7  |          | -0.9 | -0.9 | -0.9 | -0.9 | -0.9 | -0.9 | -0.9     |
| $\varepsilon^{P^*}$        | -0.1     | 0.0  | 0.3  | 0.7  | 0.7  | 1.0  | 1.0  |          | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9      |
| $\varepsilon^{CPI^*}$      | 6.8      | 7.6  | 7.7  | 8.1  | 8.6  | 8.7  | 8.8  |          | 0.4  | 0.4  | 0.4  | 0.4  | 0.4  | 0.4  | 0.4      |
| $\varepsilon^{R^*}$        | 0.1      | 0.2  | 1.2  | 3.4  | 8.1  | 9.9  | 10.2 |          | -0.1 | -0.1 | -0.1 | -0.1 | -0.1 | -0.1 | -0.1     |
| <u>Open economy shocks</u> |          |      |      |      |      |      |      |          |      |      |      |      |      |      |          |
| $\varepsilon^{\Delta S}$   | -1.8     | -2.4 | -1.6 | -1.5 | -1.4 | -1.1 | -1.1 |          | -1.3 | -1.4 | -1.4 | -1.4 | -1.4 | -1.4 | -1.4     |
| $\varepsilon^{\Delta n}$   | -2.1     | -2.9 | -3.4 | -3.6 | -3.6 | -3.6 | -3.7 |          | -2.4 | -2.8 | -3.1 | -3.4 | -3.8 | -4.5 | -5.9     |

Tab. 12: COMPARISON OF THE SHOCK DECOMPOSITION OF CONDITIONAL COVARIANCE: US-EA INTEREST RATES. The contributions of shocks sum to the conditional covariance.

| 10 <sup>-3</sup><br>quarters | DSGE-VAR |      |      |      |      |      |      | DSGE |      |      |      |      |      |      |
|------------------------------|----------|------|------|------|------|------|------|------|------|------|------|------|------|------|
|                              | 1        | 4    | 8    | 12   | 20   | 40   | ∞    | 1    | 4    | 8    | 12   | 20   | 40   | ∞    |
| <u>Cov. Cond.</u>            | 5.9      | 32.7 | 51.1 | 57.7 | 63.7 | 70.5 | 70.4 | 3.8  | 8.0  | 9.0  | 9.1  | 8.8  | 8.1  | 6.4  |
| <u>US shocks</u>             | 2.6      | 15.2 | 21.0 | 24.5 | 27.6 | 28.7 | 28.6 | 2.6  | 5.8  | 6.5  | 6.6  | 6.6  | 6.6  | 6.6  |
| $\varepsilon^A$              | 0.3      | 1.9  | 2.3  | 2.3  | 2.3  | 2.4  | 2.4  | 0.1  | 0.5  | 0.8  | 0.9  | 0.9  | 0.9  | 0.9  |
| $\varepsilon^L$              | 0.0      | 0.1  | -0.2 | 0.1  | 0.8  | 1.2  | 1.1  | 0.0  | 0.2  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  |
| $\varepsilon^I$              | 0.2      | 1.7  | 2.7  | 4.2  | 6.0  | 6.3  | 6.3  | 0.2  | 0.6  | 0.7  | 0.7  | 0.7  | 0.7  | 0.7  |
| $\varepsilon^B$              | 0.5      | 5.2  | 7.4  | 8.1  | 7.6  | 7.6  | 7.6  | 0.5  | 1.1  | 1.1  | 1.1  | 1.1  | 1.1  | 1.1  |
| $\varepsilon^G$              | 0.8      | 1.3  | 1.7  | 2.4  | 2.9  | 3.1  | 3.0  | 1.0  | 1.8  | 2.0  | 2.0  | 2.0  | 2.0  | 2.0  |
| $\varepsilon^P$              | 0.0      | 0.7  | 1.4  | 2.0  | 2.5  | 2.5  | 2.5  | 0.0  | 0.2  | 0.2  | 0.2  | 0.2  | 0.2  | 0.2  |
| $\varepsilon^{CPI}$          | 0.0      | 0.8  | 0.7  | 0.7  | 0.7  | 0.8  | 0.8  | 0.0  | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  |
| $\varepsilon^R$              | 0.8      | 3.5  | 4.9  | 4.8  | 4.8  | 4.8  | 4.9  | 0.8  | 1.3  | 1.3  | 1.3  | 1.3  | 1.3  | 1.3  |
| <u>EA shocks</u>             | 3.7      | 18.5 | 29.0 | 31.5 | 34.6 | 40.1 | 40.3 | 1.6  | 4.0  | 4.9  | 5.1  | 5.2  | 5.2  | 5.2  |
| $\varepsilon^{A^*}$          | 0.1      | -0.3 | -1.3 | -2.1 | -3.0 | -2.5 | -2.5 | 0.0  | 0.2  | 0.3  | 0.4  | 0.4  | 0.4  | 0.4  |
| $\varepsilon^{L^*}$          | 0.4      | 4.6  | 8.5  | 10.1 | 12.5 | 14.7 | 14.7 | 0.0  | 0.4  | 0.8  | 1.0  | 1.1  | 1.1  | 1.1  |
| $\varepsilon^{I^*}$          | 0.5      | 2.4  | 3.2  | 2.4  | 1.3  | 1.8  | 1.8  | 0.1  | 0.3  | 0.4  | 0.4  | 0.4  | 0.4  | 0.4  |
| $\varepsilon^{B^*}$          | 0.6      | 3.9  | 5.5  | 6.0  | 6.4  | 6.5  | 6.5  | 0.3  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  | 0.8  |
| $\varepsilon^{G^*}$          | 0.3      | 1.2  | 1.4  | 1.8  | 2.7  | 2.9  | 2.8  | 0.5  | 1.0  | 1.2  | 1.2  | 1.2  | 1.2  | 1.2  |
| $\varepsilon^{P^*}$          | 0.1      | 0.4  | 3.8  | 5.6  | 6.2  | 6.3  | 6.3  | 0.1  | 0.2  | 0.3  | 0.3  | 0.3  | 0.3  | 0.3  |
| $\varepsilon^{CPI^*}$        | 0.1      | 1.5  | 1.7  | 1.7  | 1.9  | 2.2  | 2.3  | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  |
| $\varepsilon^{R^*}$          | 1.7      | 4.8  | 6.1  | 6.0  | 6.5  | 8.2  | 8.4  | 0.5  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  | 0.9  |
| <u>Open economy shocks</u>   |          |      |      |      |      |      |      |      |      |      |      |      |      |      |
| $\varepsilon^{\Delta S}$     | -0.6     | -2.1 | -0.5 | 0.2  | 0.0  | 0.0  | -0.1 | -0.7 | -2.1 | -2.6 | -2.6 | -2.7 | -2.7 | -2.7 |
| $\varepsilon^{\Delta n}$     | 0.2      | 1.1  | 1.6  | 1.5  | 1.6  | 1.7  | 1.5  | 0.3  | 0.4  | 0.2  | 0.0  | -0.3 | -1.0 | -2.7 |

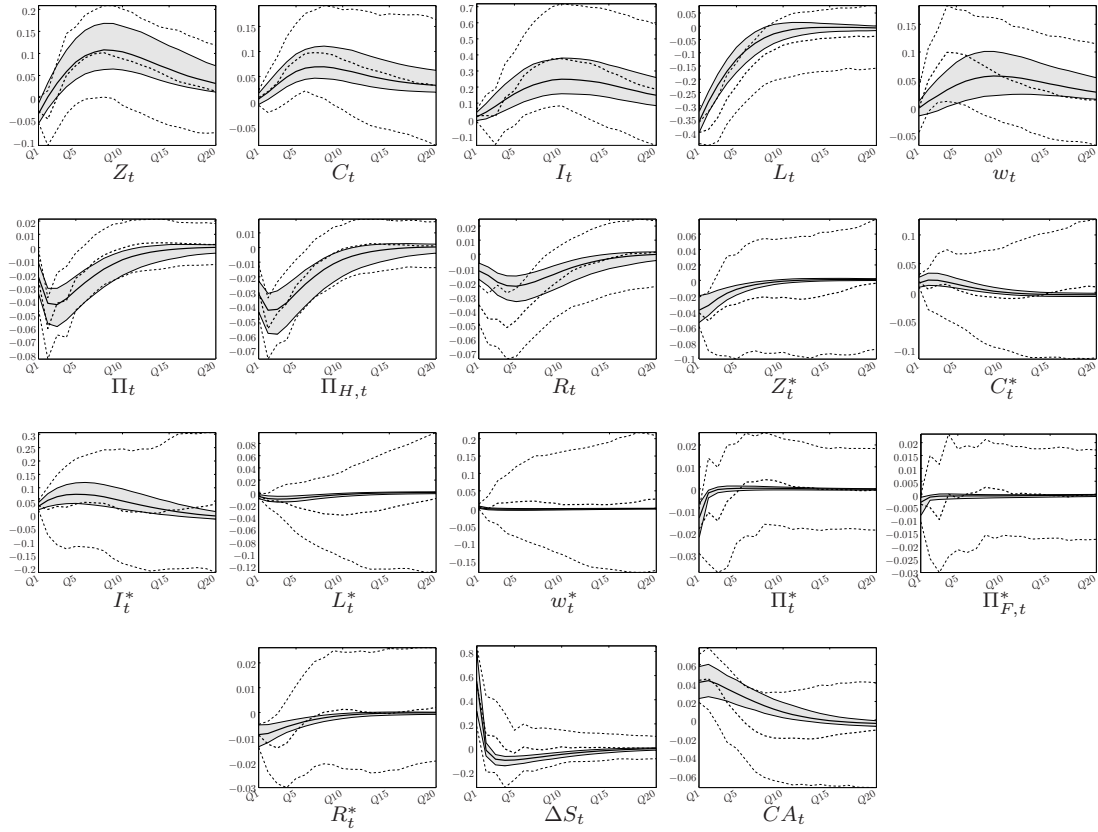


Fig. 4: Impulse Response Functions associated to a shock on  $\epsilon_t^A$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

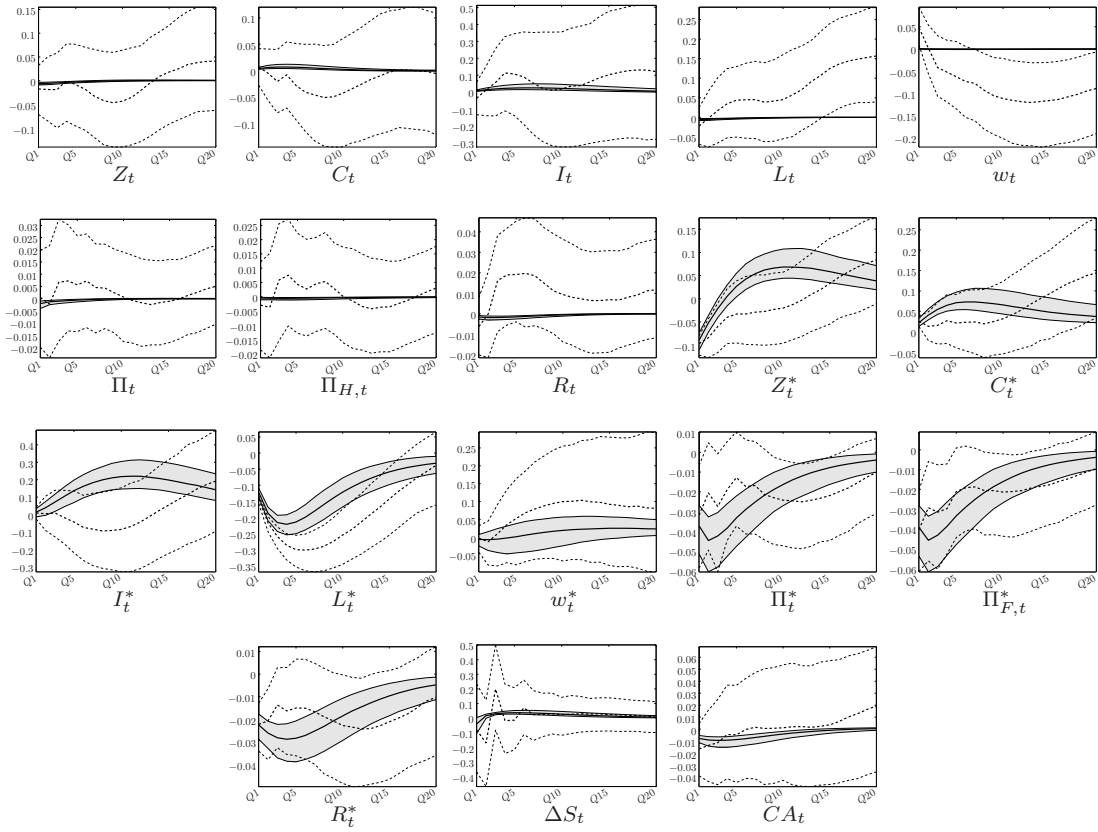


Fig. 5: Impulse Response Functions associated to a shock on  $\epsilon_t^{A^*}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

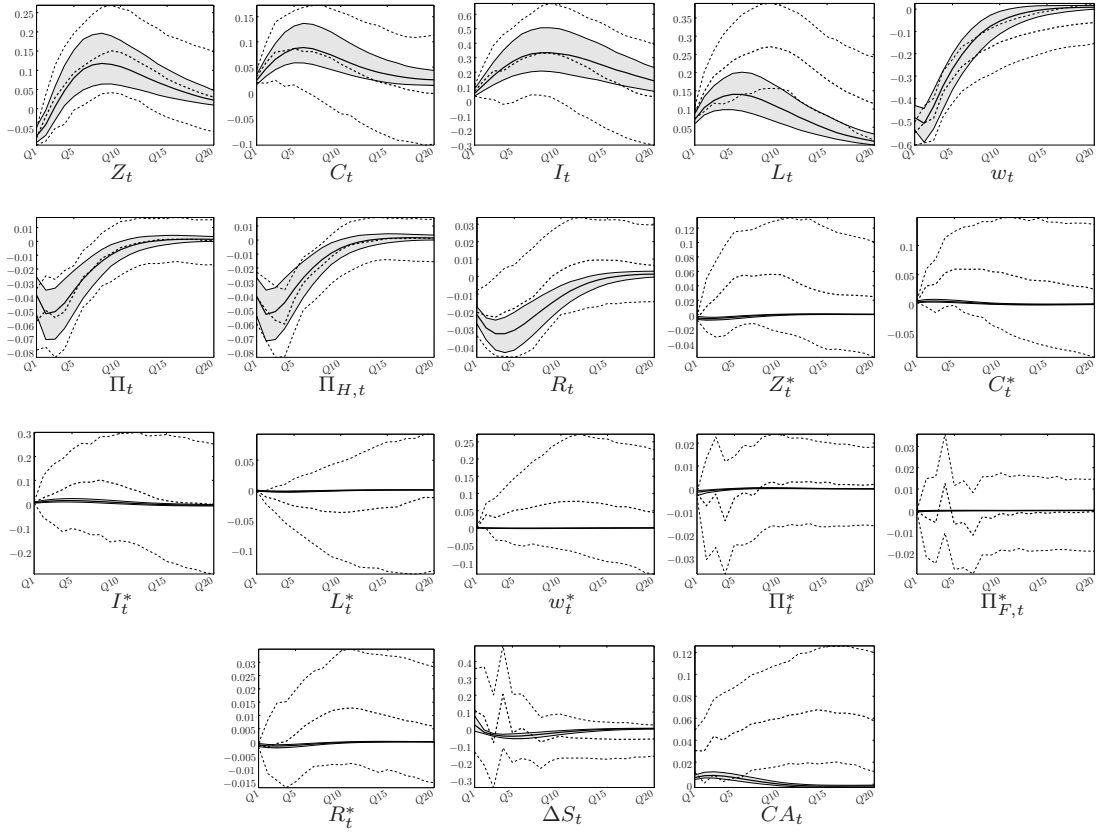


Fig. 6: Impulse Response Functions associated to a shock on  $\epsilon_t^L$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

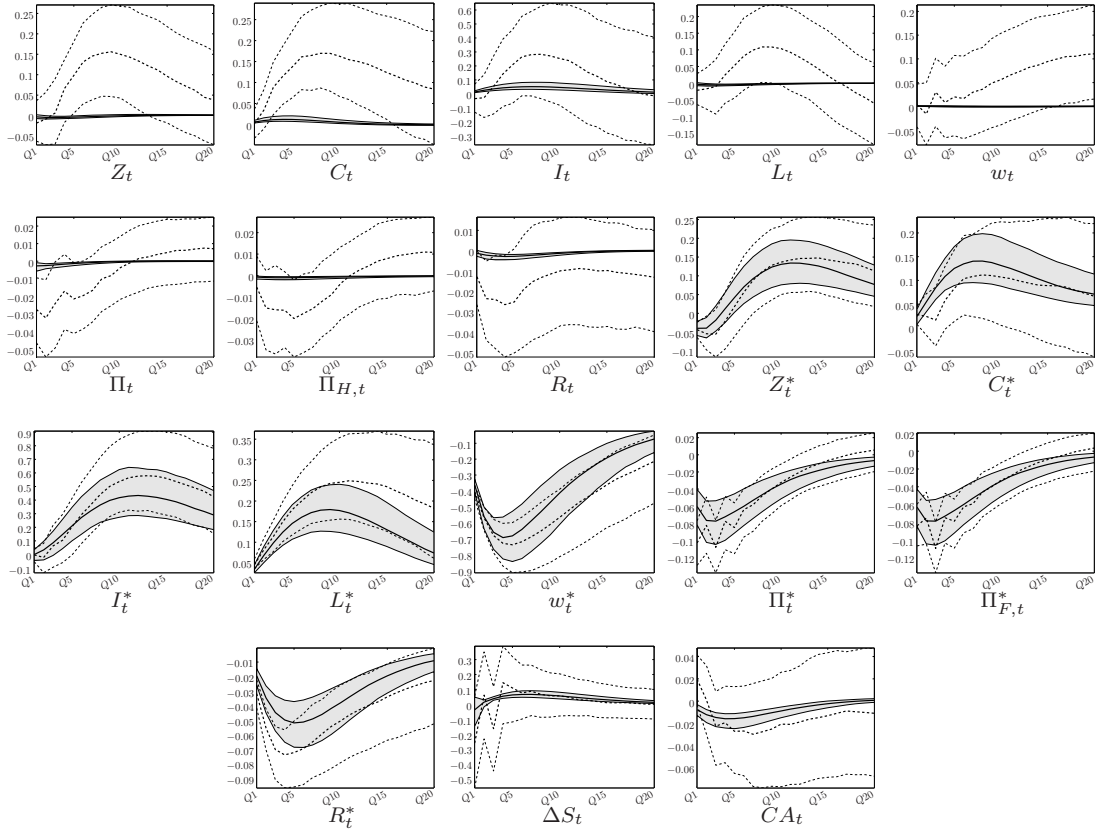


Fig. 7: Impulse Response Functions associated to a shock on  $\epsilon_t^{L^*}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

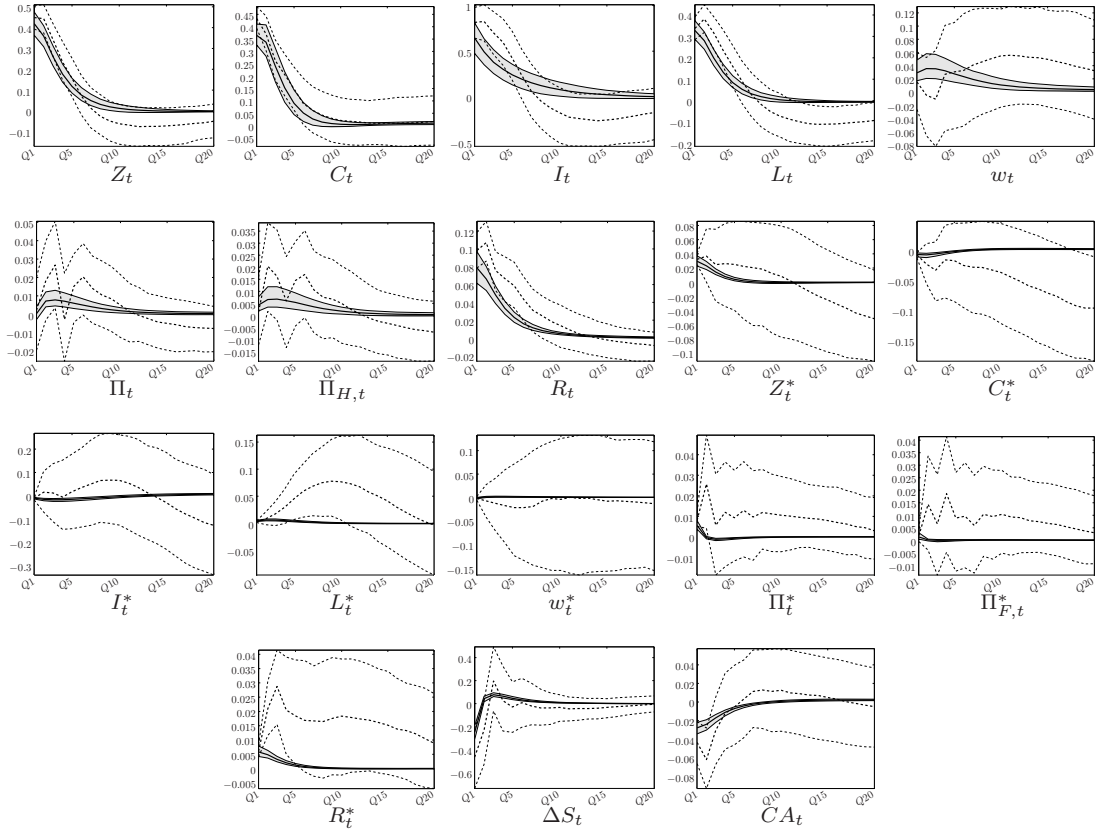


Fig. 8: Impulse Response Functions associated to a shock on  $\epsilon_t^B$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).



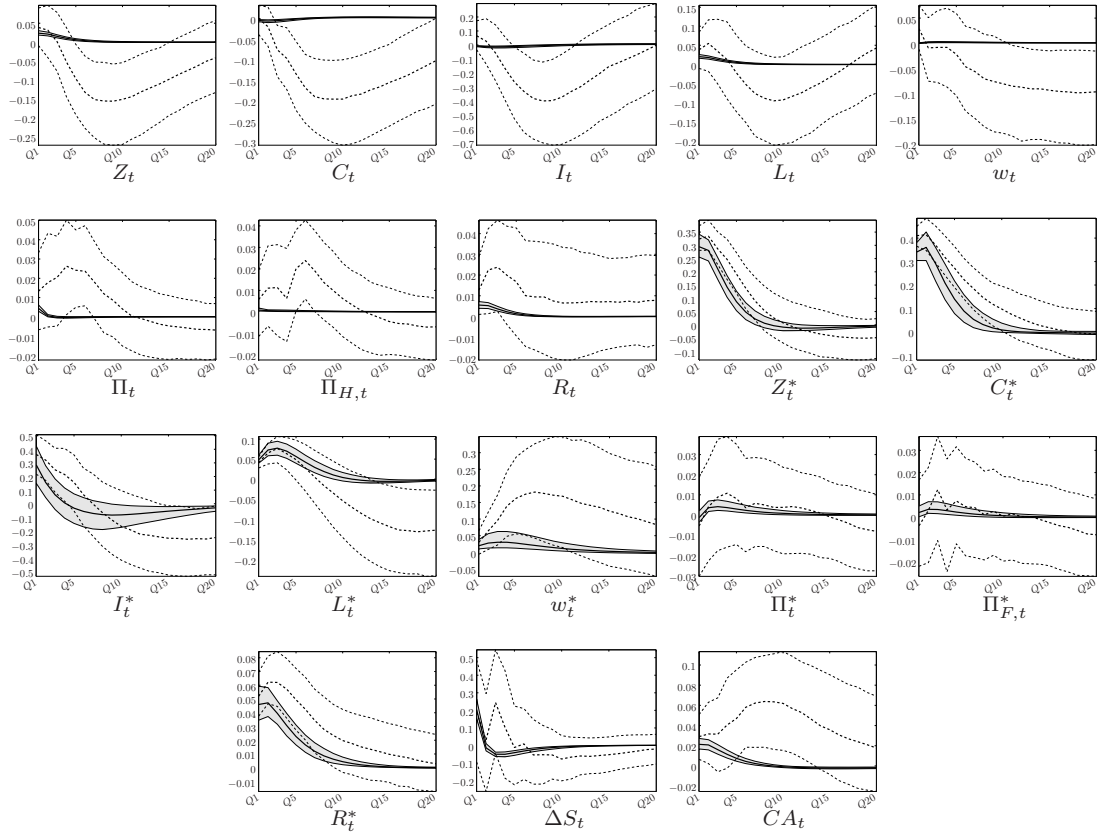


Fig. 9: Impulse Response Functions associated to a shock on  $\epsilon_t^{B*}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

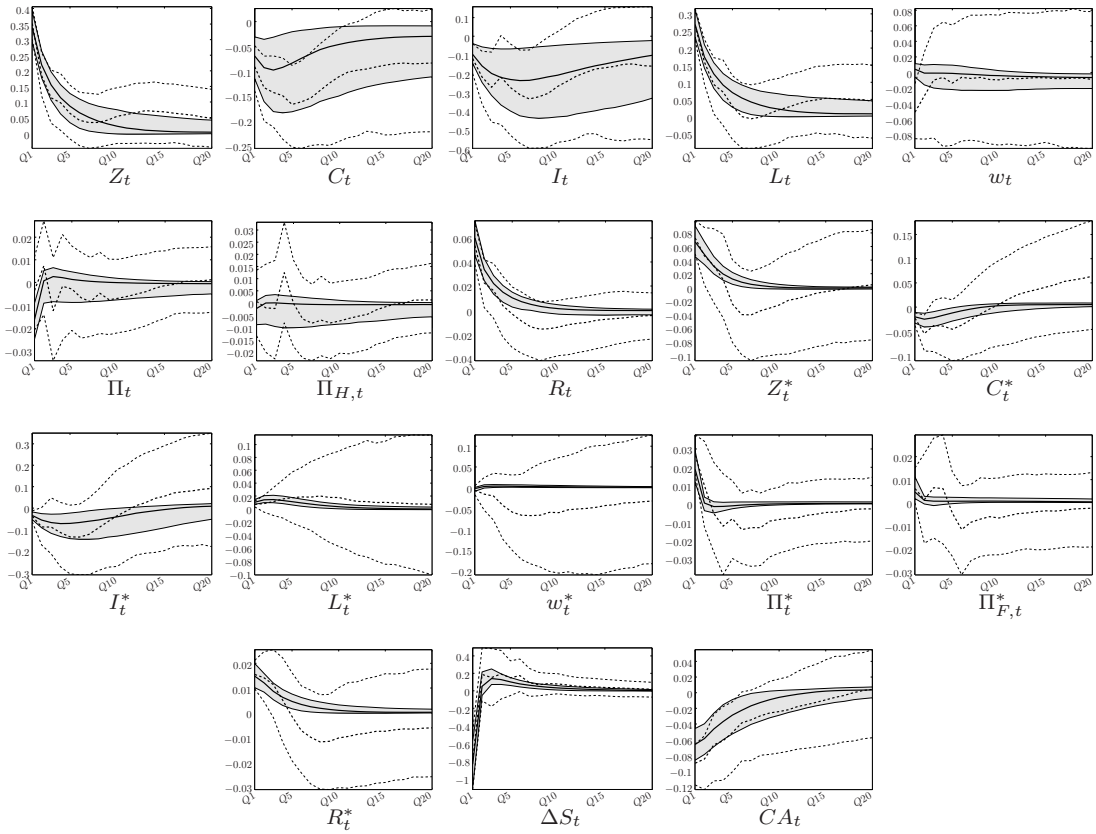


Fig. 10: Impulse Response Functions associated to a shock on  $\epsilon_t^G$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

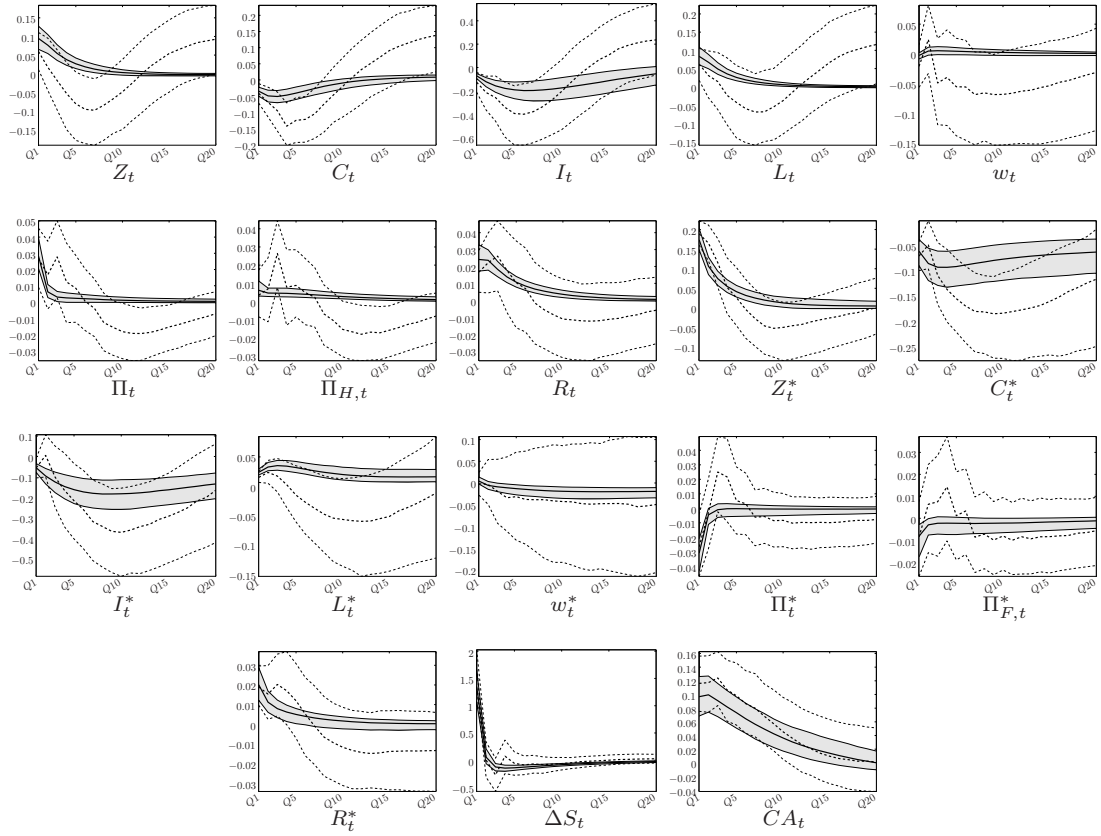


Fig. 11: Impulse Response Functions associated to a shock on  $\epsilon_t^{G^*}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

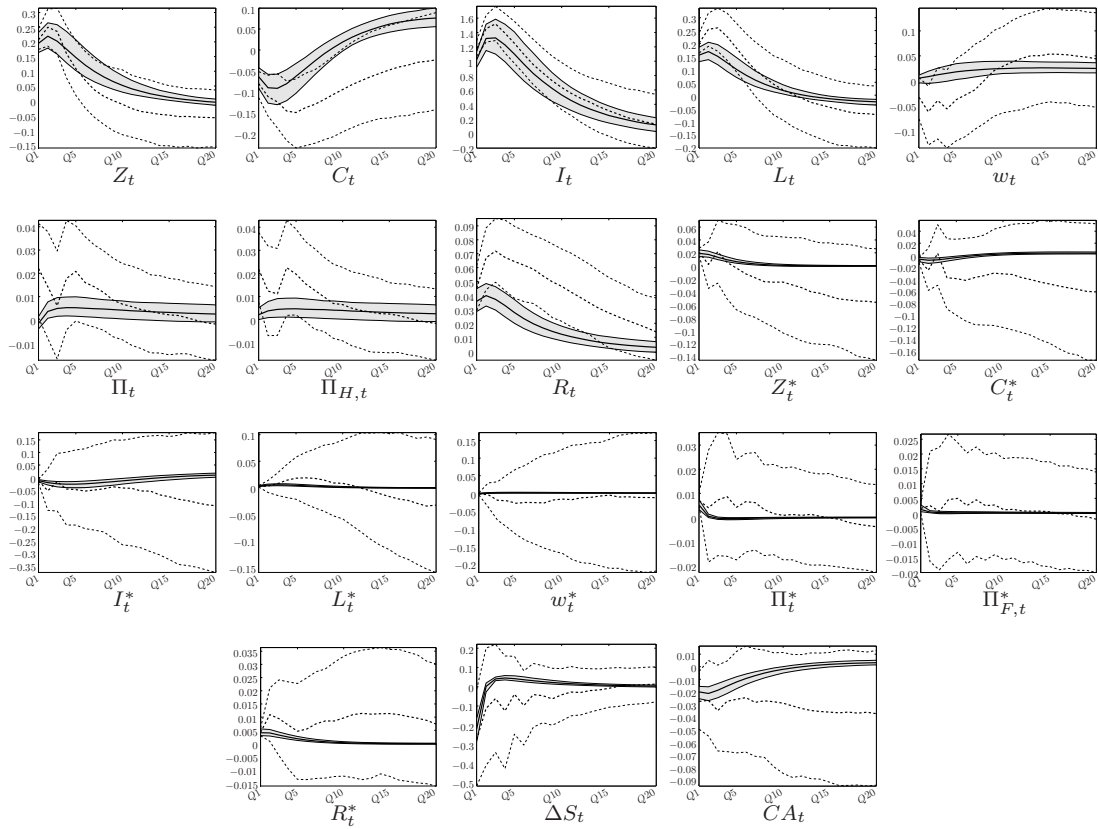


Fig. 12: Impulse Response Functions associated to a shock on  $\epsilon_t^I$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

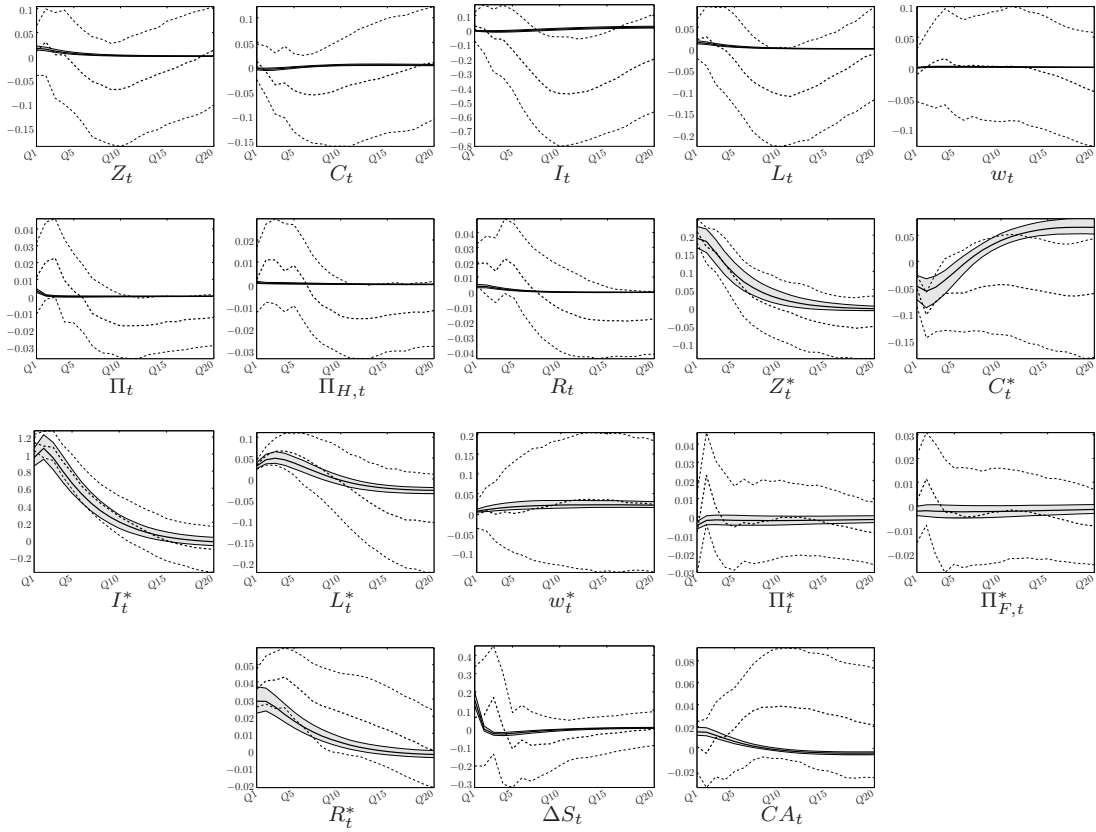


Fig. 13: Impulse Response Functions associated to a shock on  $\epsilon_t^{I^*}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

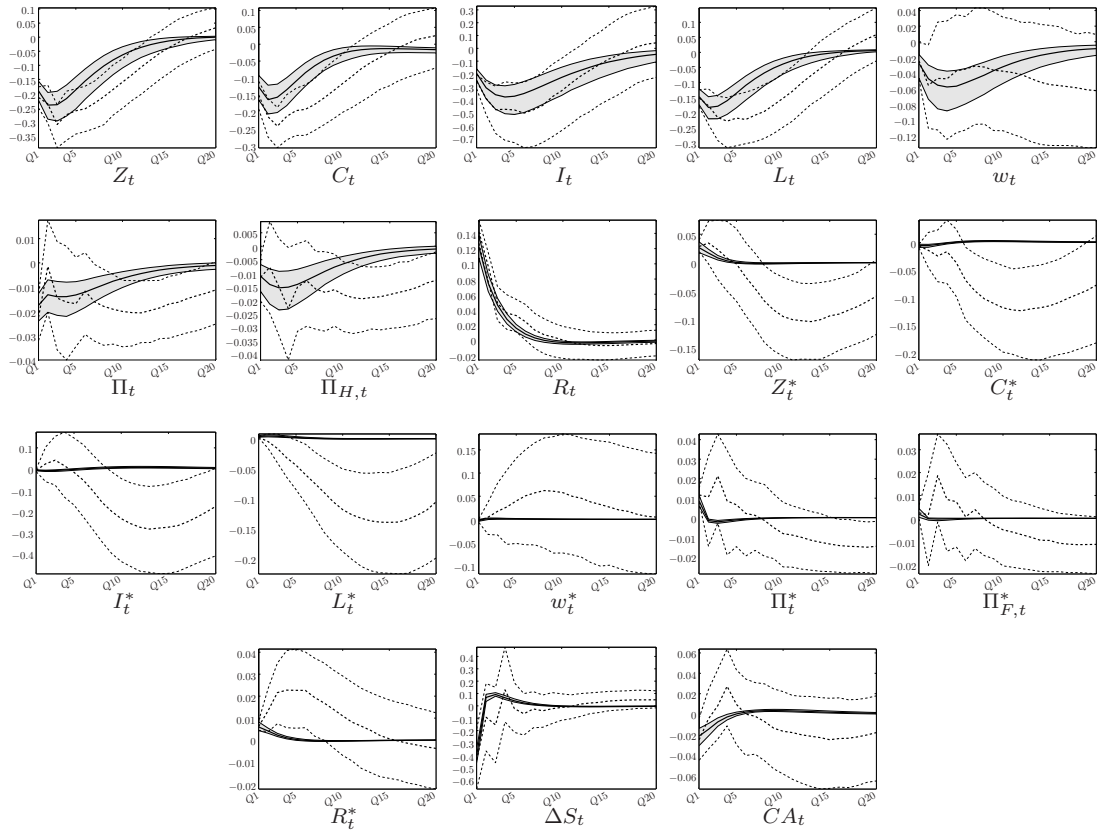


Fig. 14: Impulse Response Functions associated to a shock on  $\epsilon_t^R$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

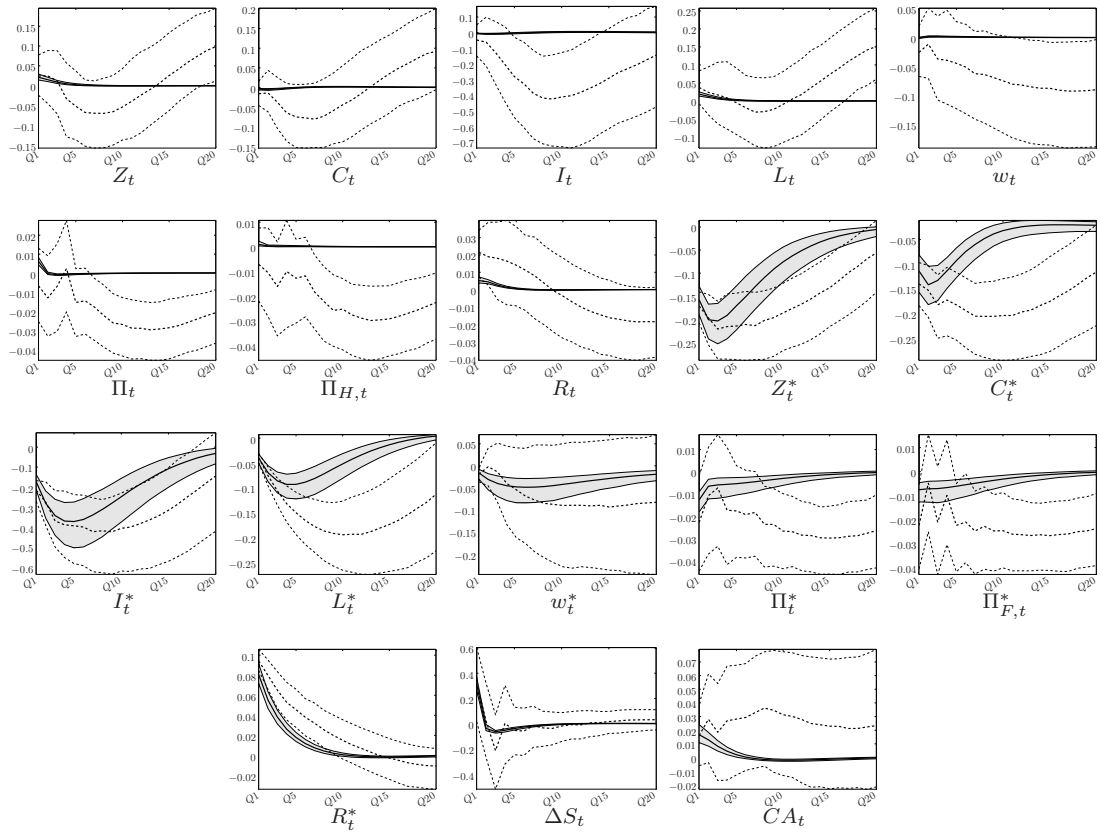


Fig. 15: Impulse Response Functions associated to a shock on  $\epsilon_t^{R^*}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

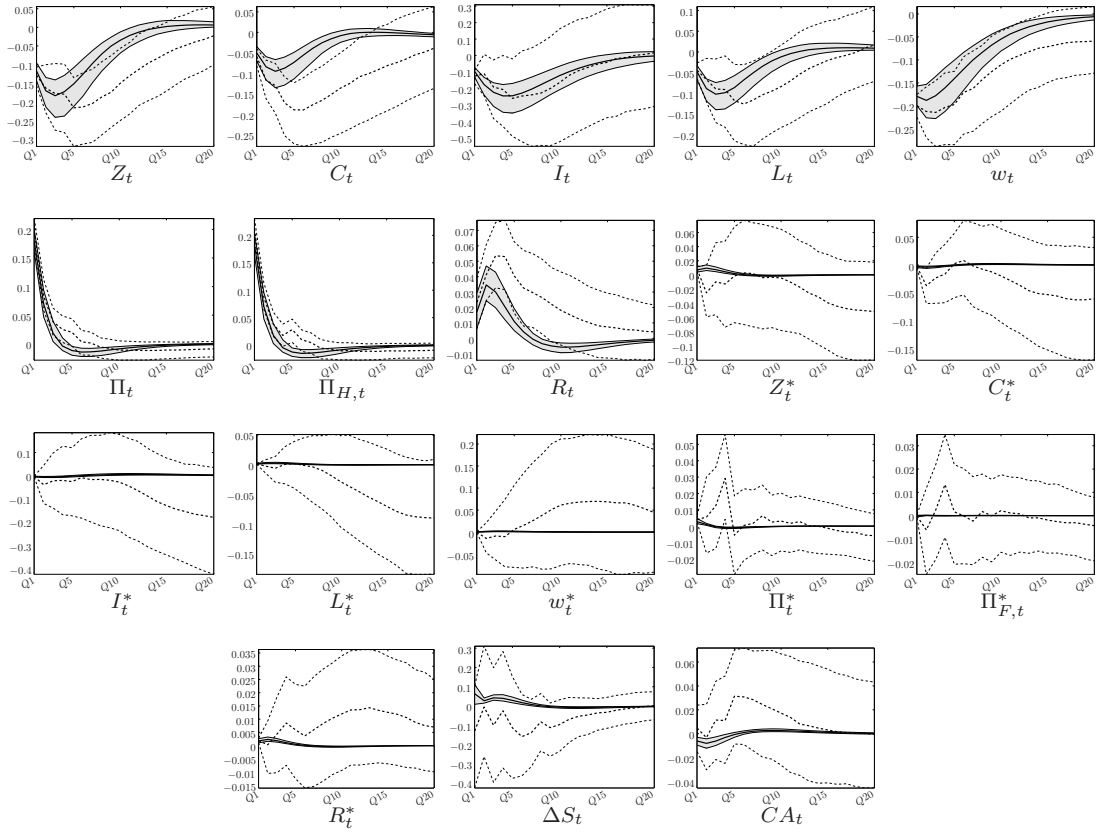


Fig. 16: Impulse Response Functions associated to a shock on  $\epsilon_t^P$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).



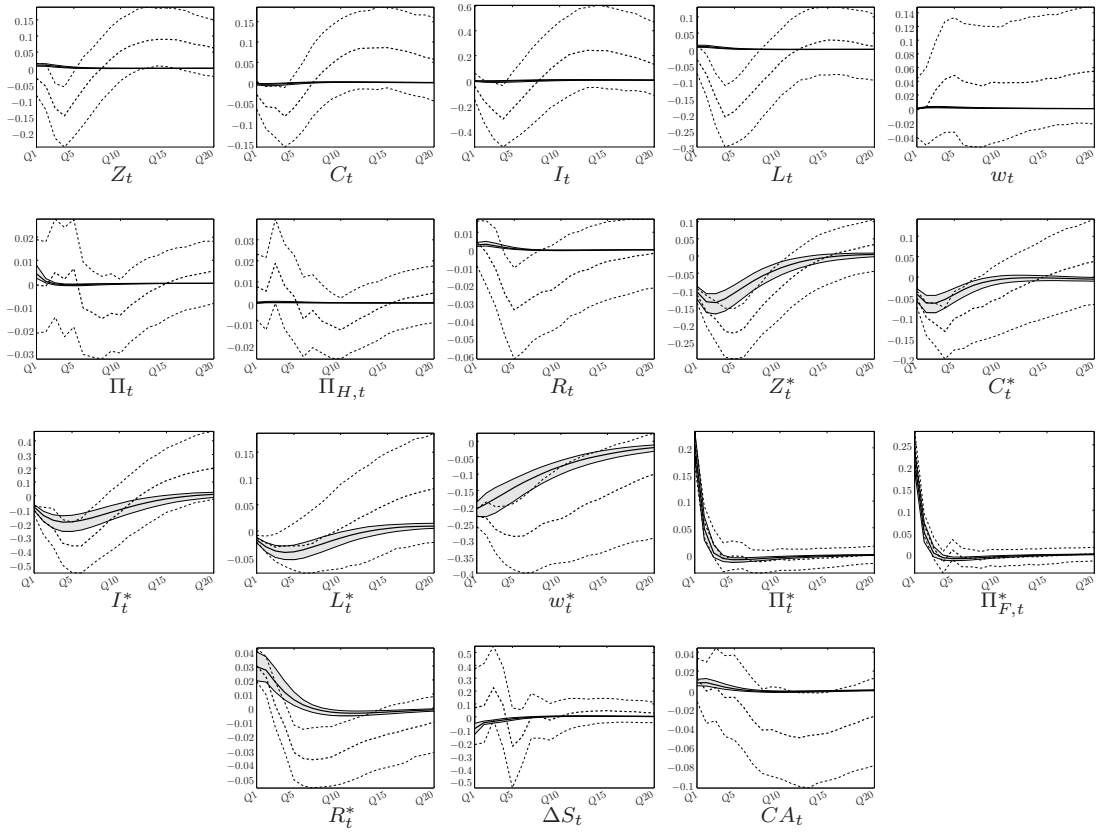


Fig. 17: Impulse Response Functions associated to a shock on  $\epsilon_t^{P^*}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

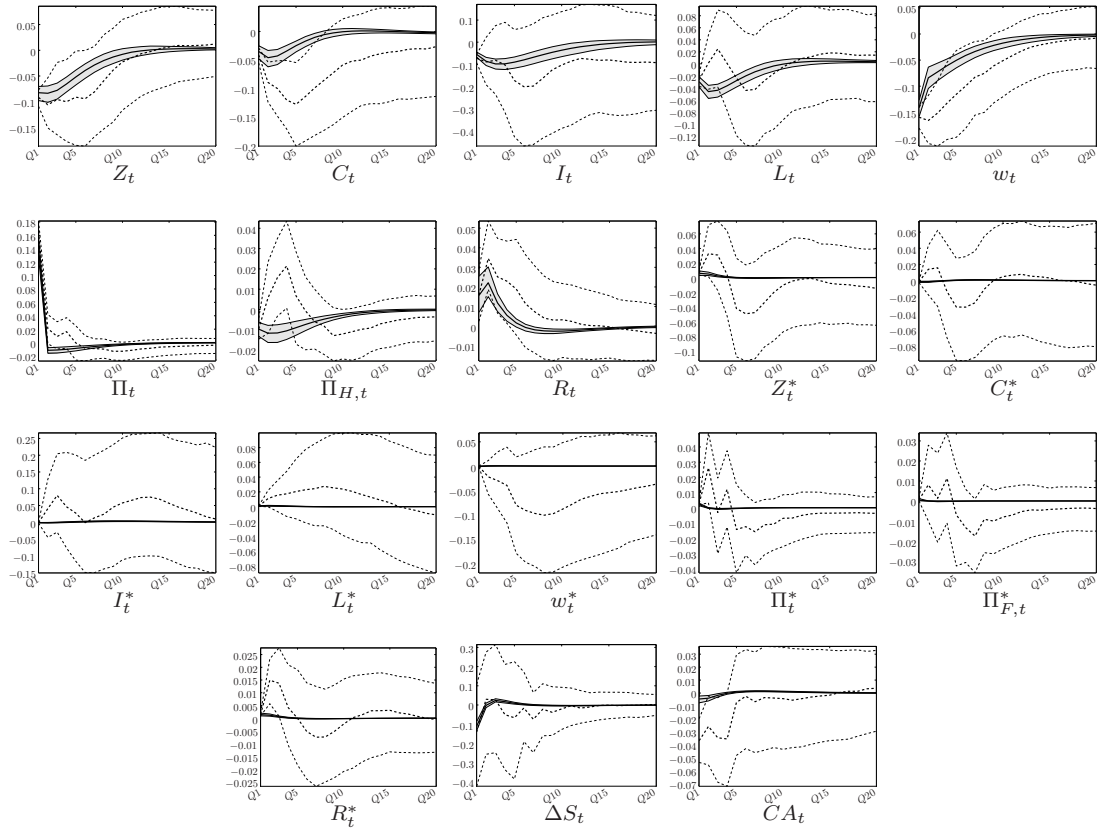


Fig. 18: Impulse Response Functions associated to a shock on  $\epsilon_t^{CPI}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

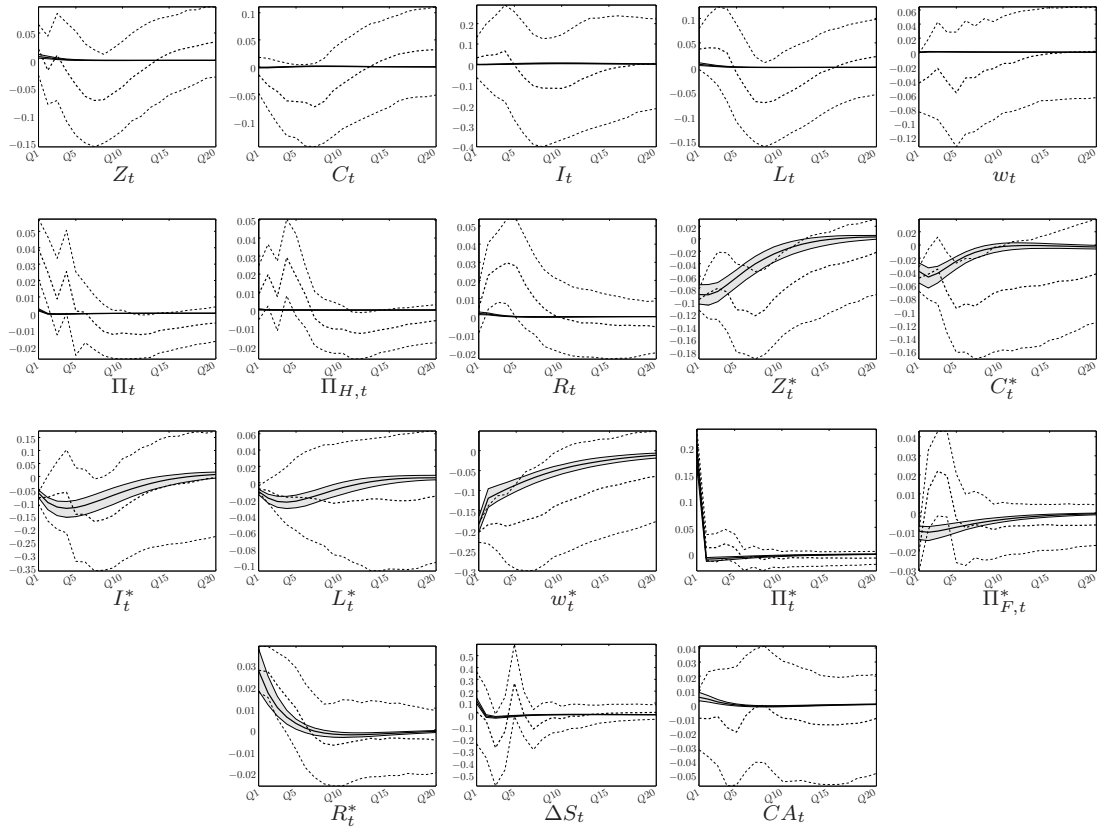


Fig. 19: Impulse Response Functions associated to a shock on  $\epsilon_t^{CPI^*}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

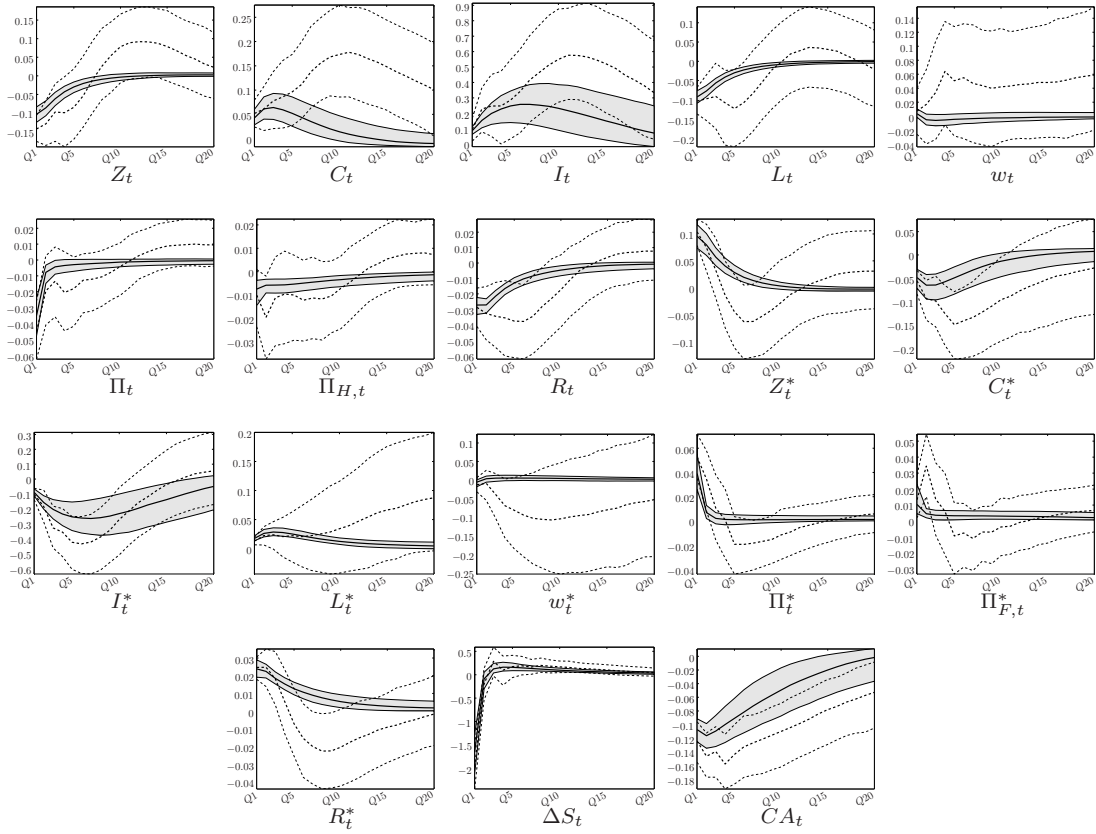


Fig. 20: Impulse Response Functions associated to a shock on  $\epsilon_t^{\Delta S}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

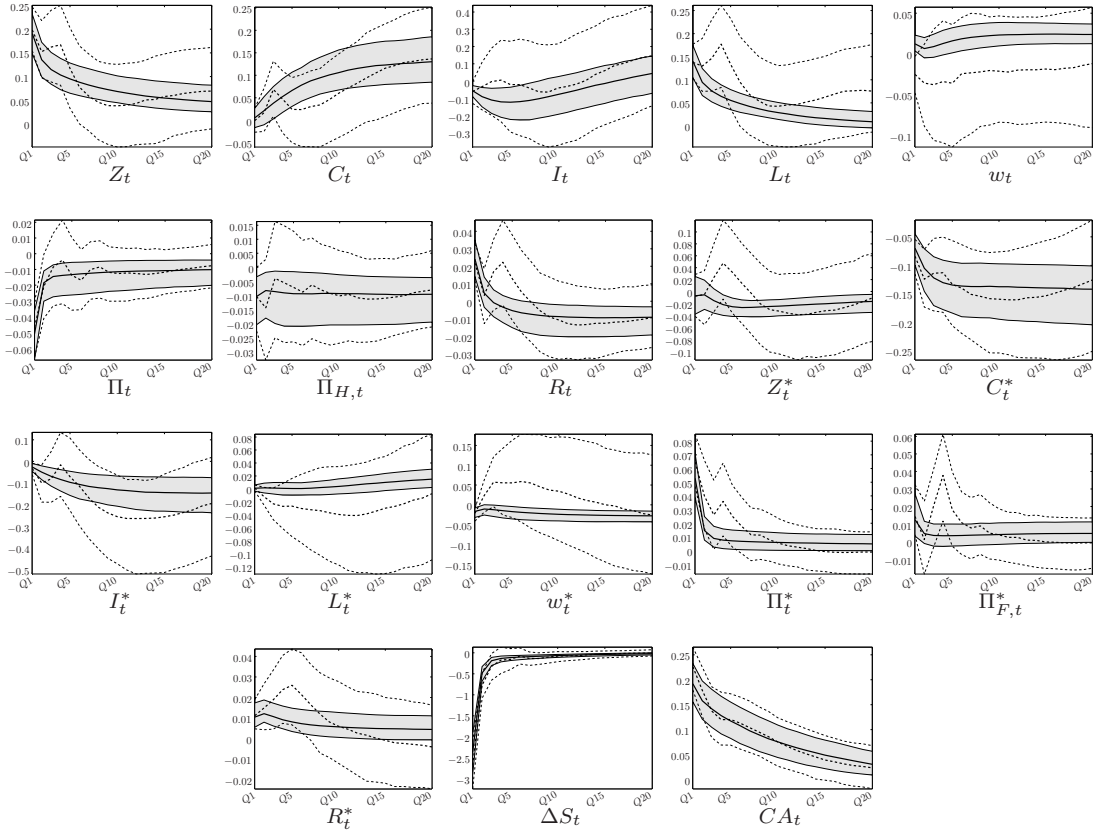


Fig. 21: Impulse Response Functions associated to a shock on  $\epsilon_t^{\Delta n}$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

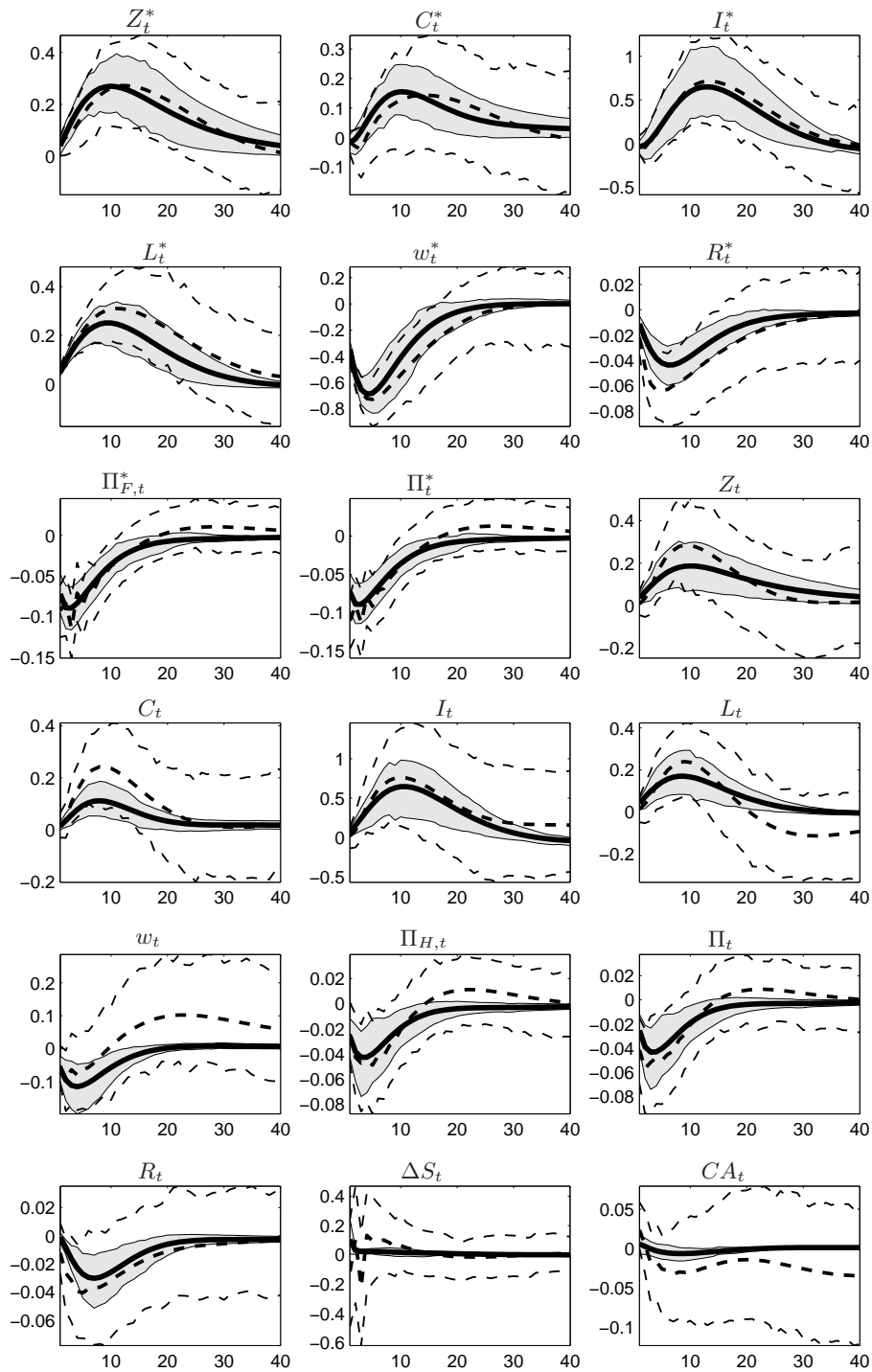


Fig. 22: Impulse Response Functions associated to a shock on  $F_t^L$ . DSGE-VAR (dotted lines), DSGE (plain lines and shaded areas).

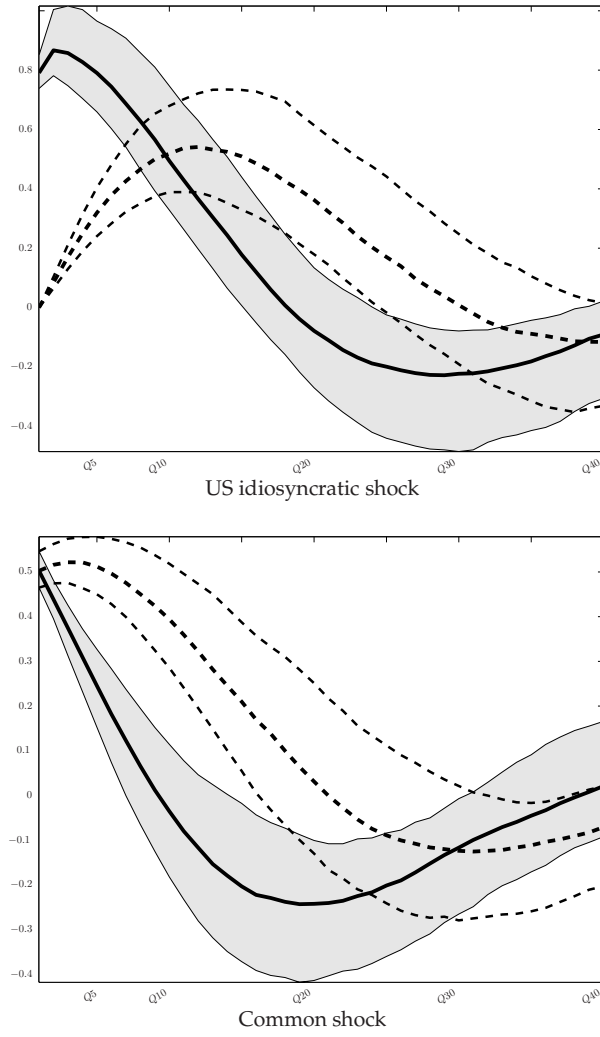


Fig. 23: Impulse Response Functions of US and euro area GDP associated to an idiosyncratic US shock and a common shock. *Euro area (dotted lines), US (plain lines and shaded areas).*